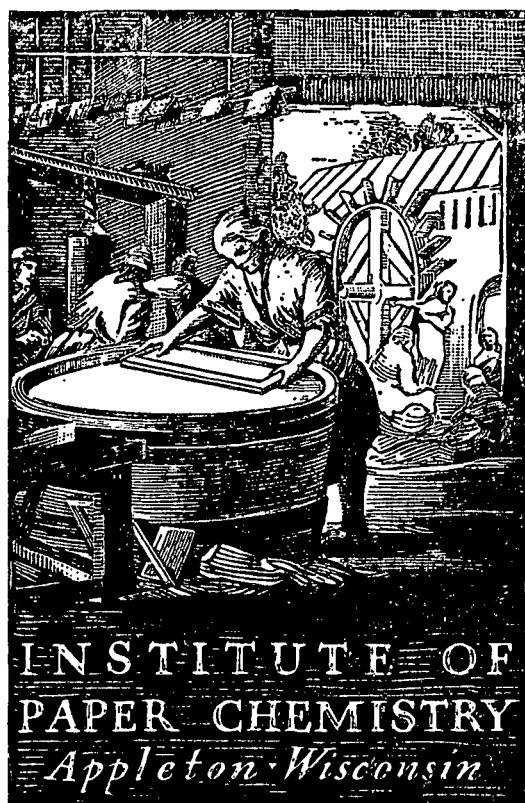


Whitcomb



**RELATIONSHIP BETWEEN SACK PERFORMANCE AND
THE PROPERTIES OF SACK PAPER
PART III. VERIFICATION OF A FATIGUE LIFE THEORY
FOR REGULAR SACK PAPER**

Project 2033

Report Nineteen

**A Progress Report to
MULTIWALL SHIPPING SACK
PAPER MANUFACTURERS**

October 9, 1961

THE INSTITUTE OF PAPER CHEMISTRY

Appleton, Wisconsin

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TABLE OF CONTENTS

	Page
SUMMARY	1
INTRODUCTION	5
MATERIALS	11
TEST PROCEDURE	13
Virgin Tensile Test	14
One-Cycle Tensile Test	14
Fatigue Life Tests	15
CALCULATION OF THEORETICAL FATIGUE LIFE	18
Approximation to the Virgin Load-Elongation Curve	20
Approximation to the Reload Curves	26
Example of Calculation of a Theoretical Fatigue Life	31
DISCUSSION OF RESULTS	35
Experimental Fatigue Life	35
Comparison of Theoretical and Experimental Fatigue Life	44
Comparison of Total Stretch and Virgin Stretch	53
PROPOSALS FOR FUTURE WORK	66
Further Verification of Theory	67
Improvement in Accuracy of Theory	68
Extension of Fatigue Theory to Other Repeated Tension Processes	69
Simplification of Fatigue Life Equation	69
LITERATURE CITED	71

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SUMMARY

1. Previous investigations indicate that the repeated impact performance of multiwall sacks depends on the tensile fatigue characteristics of the sack paper, among other factors.

2. Fatigue performance of a material or of a structure is conventionally described by its fatigue life, which is defined as the number of applications of stress or strain which can be sustained before failure.

3. A preceding report described a mathematical theory which has been developed for uniaxial fatigue performance of sack paper. The theory provides an equation for estimating the tensile fatigue life of sack paper in terms of (a) its virgin load-elongation properties, (b) a property relating to its reload behavior in repeated tension, and (c) the magnitude of strain applied repetitively to the paper.

4. The prospective usefulness of the mathematical theory of fatigue is that the fatigue life of sack paper may be predicted from a knowledge of its properties in simple tension, avoiding thereby the necessity of performing a fatigue life test. Perhaps even more important, the effect of fiber and processing variables which are reflected in the simple tension test can be translated into potential fatigue performance by means of the theory.

5. The purpose of the present study is to verify experimentally the theoretical equation for fatigue life.

6. Fatigue life tests were performed on two samples of regular, 50-lb. kraft sack paper in both principal directions at various levels of applied strain. These samples represented both average and extreme properties for regular sack paper with respect to virgin stretch and tensile energy absorption.

7. The relationship between observed fatigue life and applied strain was found to be highly nonlinear in the range of fatigue life comparable to sack impact fatigue life, namely, zero to 20 cycles.

8. The variability in experimental fatigue life was very high, particularly when the applied strain was near the endurance limit (the applied strain at which the fatigue life theoretically becomes infinitely large). It was not uncommon for the fatigue lives of specimens within a sample to range over nearly the entire range of fatigue lives investigated, i.e., from zero to 20 cycles.

9. Keeping in mind that the limited sampling of this study involved wide differences in paper properties, tensile fatigue life ranked the samples compatibly with their sack impact performance and their Frag impact fatigue performance.

10. In general, theoretical and experimental fatigue life agreed to within a few cycles for moderately low fatigue lives. The theory usually underestimated the fatigue life and is attributable, at least in part, to

conservatism purposely introduced in the application of the theory. While numerical differences between theoretical and experimental fatigue life were quite large near the endurance limit, there appeared to be reasonable agreement between the theoretical and the apparent endurance limit (strain).

Further study is required to determine whether the endurance limit has utility as an index of potential performance. In one case, discrepancies between theory and experiment are believed to be explainable by sampling variability.

11. In all cases the theoretical curve of fatigue life vs. applied strain approximated closely the nonlinear shape of the experimental curve, indicating that the theory accounts for this essential characteristic of the fatigue behavior of sack paper.

12. The experimental endurance limit was found to be substantially greater than the proportional limit strain--a result which was predicted by the theory.

13. Application of the fatigue theory requires determination of the tension reload characteristics and approximation of the virgin tensile load-elongation curve by straight lines. Practical procedures have been devised for determining these properties.

14. A critical assumption in the fatigue theory is the invariancy of the total stretch; that is, it is assumed that the total stretch available in the paper is independent of the number of cycles of repeated tension and is equal to the virgin stretch. Examination of the experimental data gives no

cause to doubt the validity of this assumption as a first and reasonable approximation. But stretch (virgin and total) is so highly variable that appreciable errors can be incurred in theoretical predictions of fatigue life when sampling is small.

15. In view of (a) the high variability in fatigue phenomena and (b) the scant state of theoretical knowledge concerning sack paper fatigue, it is the opinion of the investigators that the agreement between theory and experiment may be regarded as generally favorable. It is believed that further development of the theory of fatigue performance will prove to be profitable to sack paper technology.

INTRODUCTION

The laboratory repeated impact performance of multiwall sacks appears to be dependent on the fatigue and energy absorption characteristics of the sack paper from which they are fabricated (in addition to design, fabrication efficiency and commodity characteristics). That fatigue considerations should play an important role in laboratory sack impact behavior is really self-evident inasmuch as the failure of a material after repeated applications of a stress or strain is, by definition, a fatigue phenomenon. Although the results of a laboratory sack impact test are more commonly expressed in terms of accumulated safe inches of drop, the essence of the test is the determination of the number of safe drops sustained before rupture of the sack. The latter measure is the conventional definition of fatigue life.

The importance of the energy absorption characteristics of the sack paper is indicated from both (a) the concept of the mechanics of sack impact and (b) the results of correlation studies of sack paper properties and sack performance. During impact a portion of the kinetic energy of the sack and contents is dissipated in the form of strain energy of the sack paper associated with distention of the sack as the contents "explode" outward against the sack walls. Inasmuch as the sack is flexible, its distention (and the accompanying energy absorption) must be largely tensile. Correlation studies tend to substantiate the aforementioned description of sack impact inasmuch as the better correlations of sack performance and paper properties have involved tensile energy absorption stretch (which is closely allied to energy), Van der Korput dynamic energy, impulse, Frag and Thwing-Albert impact fatigue (the latter two tests being fatigue tests with a constant applied energy) (1).

Taken together, the tensile fatigue and tensile energy absorption characteristics of sack paper comprise an area of sack paper technology about which relatively little is known in a fundamental sense. The basic mechanism of fatigue rupture appears to be a progressive deterioration in the ability of the paper to absorb energy with each succeeding application of stress and strain; eventually, the energy absorption capacity of the paper is reduced to such a level that the following application exceeds the paper's residual energy level and rupture occurs. The number of cycles prior to rupture is by definition the fatigue life of the paper.

The complicating factor in this mechanism is that during the unloading phase of each cycle of stress or strain the paper recovers some of its capacity to absorb energy, both instantaneously during unloading and during whatever time period may exist between cycles. Were it not for this recovery, the determination of fatigue life would be a trivial problem--simply the ratio of the virgin energy absorption and the applied energy (in the case of a constant applied energy). Because of the recovery, however, the problem becomes exceedingly more complex. The recovery characteristics of the paper must be taken into account and it has been shown that these characteristics vary nonlinearly with the number of prior stress applications (2) and probably vary between papers (3).

A homely analogy may serve to illustrate the intrinsic nature of fatigue behavior of sack paper. Suppose that water is being removed from a pail by means of a dipper. Each dip removes a quantity of water and eventually the content of the pail will be exhausted. The number of dips required to empty the pail is analogous to fatigue life. Now consider the additional factor that

the dipper has a hole in its bottom, so that during dipping some of the water returns to the pail. The question is: how many dips are required to empty the pail? With no hole in the dipper the answer is readily determined; merely divide the original quantity of water in the pail (analogous to virgin tensile energy absorption) by the capacity of the dipper (analogous to applied energy). But with the defective dipper, the determination is no longer so simple. Obviously, further consideration must be given to the size of the hole and the rapidity of the dipping operation. Without pursuing the analogy unduly, it may be appreciated that the fatigue life of sack paper requires consideration of other factors peculiar to repeated tension behavior in addition to virgin tensile energy absorption and applied work.

It is believed that considerable progress has been made toward gaining an understanding of the behavior of sack paper in repeated uniaxial tension and expressing theoretically the behavior in mathematical terms. The latter effort (3) resulted in an explicit equation relating fatigue life to (a) virgin tensile properties of the sack paper, (b) a reloading characteristic of the paper (the shape of the second and succeeding load-elongation curves in repeated tension differ somewhat from the virgin curve), and (c) the magnitude of the applied stress and strain.

It might be noted that, in general, a fatigue tester is capable of relating fatigue life to only the latter of the above-mentioned factors, that is, applied stress or strain. The usefulness of the fatigue life theory, therefore, resides in the prospect that fatigue life of a given sample of sack paper may be predicted from a knowledge of certain basic tension characteristics which are capable of being evaluated in any laboratory possessing modern tensile testing

equipment. Moreover, the effect of fiber properties and papermaking variables on fatigue performance may be studied, to the extent that they are discernible in the results of the conventional tensile test. For example, several current pioneering efforts in paper research are directed to relating fiber properties and web structure to the elastic and plastic behavior of the sheet in tension (4,5). If this type of knowledge could be projected, in turn, to potential fatigue life performance (as is the intent of the fatigue theory), the paper-maker would have available a much better understanding of the relationship between his raw materials and processes, on the one hand, and its capabilities after conversion into a multiwall sack.

It should be remarked that as of the present time the theoretical work has been performed for the case of a constant elongation (or constant unit strain) repetitively applied to sack paper in a uniaxial tension tester. An applied elongation process was selected for study over an applied energy process because the former is somewhat easier to visualize and analyze and, moreover, is closely related to applied energy. There is no major difference in the underlying principles of the two types of repeated tension processes, however, and it is expected that an equation for fatigue life under repeated applied energy can be derived paralleling the one for applied elongation.

It is anticipated that improvements in the understanding of the relationship between sack performance and sack paper properties can be afforded by considering the biaxial fatigue characteristics of the paper, inasmuch as the sack walls are subjected to biaxial stresses during impact. But unless uniaxial fatigue behavior of paper is well understood, there seems to be small

prospect of understanding its biaxial fatigue behavior--beyond that gained from direct fatigue life testing (for which, incidentally, a controllable biaxial tester does not now exist in the industry).

This report concerns the experimental verification of the fatigue life theory presented in the preceding report of this series (3). Verification involves (a) testing sack paper samples in repeated tension to find their fatigue lives for various levels of applied strain, and (b) determination of the parameters required in the theory, i.e., the virgin tensile and reload characteristics of the samples, (c) calculation of theoretical fatigue life from the data of (b) for the applied strains of (a), and (d) comparison of theory and experiment.

It may be recalled that verification of the fatigue theory can also be studied by comparing the theoretical and experimental residual stretch as a function of the number of cycles (3). This method of verification may be regarded as an alternative to that outlined above and is left for a future report as interest may warrant.

Finally, it should be mentioned that work is now going forward to determine the relationship between Instron fatigue life in one or both principal directions of the paper and the progressive height drop performance of regular and extensible multiwall sacks from the current fabrication program. Initially, the interest in repeated tension was motivated by the observation that fatigue type tests such as Frag or Thwing-Albert were correlated quite well with sack performance and it was desired to study fatigue behavior under the readily controllable conditions of a tensile test. Now that considerable

progress has been made in the theory of uniaxial tensile fatigue, a study of its relationship to sack performance is timely. It should be noted that Mappus (6) found that the Instron fatigue lives of several samples of regular and extensible sack papers (under conditions of constant applied energy) were ranked compatibly with the performance, in general terms, of these two types of sack constructions.

MATERIALS

For this experimental study two samples of regular 50-lb. sack paper were selected from the butt rolls remaining from the 1957 fabrication program (1). The samples correspond to the papers used in the middle ply of the "end" samples of the pasted sacks of Runs R and T. The tension, tear, and impact fatigue properties of these samples are listed in Table I. The tensile, stretch, and work properties shown in Table I were obtained during the course of the present study, whereas the tear and impact fatigue data are extracted from Appendix A of Reference (1).

Comparison of the data in Table I of this report and Table XII of Reference (1) reveals that Run R-2 is an "average" sample relative to the entire collection of fabrication program materials, with the exception of cross-machine stretch and tensile work which tend to be slightly lower than average. Run T-2, on the other hand, possesses extreme tension properties with respect to the other fabrication samples. The in-machine stretch and tensile work are near the minimum values reported in Reference (1) while the cross-machine stretch and work are near the maximums. Thus, although the present study was limited to two samples, they represent both the average and the extreme tension properties which may be expected in regular 50-lb. sack papers.

TABLE I
PHYSICAL CHARACTERISTICS OF SACK PAPER SAMPLES

Roll No.	Direction	Tensile, lb./in.	Stretch, %	Tensile Work, in.-lb./sq.in.	Elmendorf Tear, g./sheet	Thwing-Albert Impact Fatigue, falls to failure ^a	Frag Test, falls to failure
R-2	In	31.2	1.83	0.376	123	} 26 {	29
R-2	Cross	18.8	2.57	0.359	134		52
T-2	In	31.8	1.33	0.273	124	} 24 {	12
T-2	Cross	15.4	5.49	0.650	141		89

^a Thwing-Albert impact fatigue is a biaxial test.

TEST PROCEDURE

Three types of uniaxial tensile tests were performed in this study: (a) virgin tensile test, (b) single-cycle tensile test, and (c) fatigue life at a constant applied elongation (i.e., repeated tension). The virgin and single-cycle tests provided data on the several physical properties required for the theoretical prediction of fatigue life; the repeated tension tests gave experimental data on fatigue life for comparison with the predicted values. Several aspects of the sampling and testing procedure were common to two or more of the three types of tests and, therefore, are conveniently described in general.

Ten specimens were prepared and tested for each test condition. The ten specimens in each sample were procured at two-foot intervals along the machine direction of the butt roll. After standard conditioning the specimens were cut to one-inch width and six-inch span. All testing was performed in a Baldwin-Southwark Universal testing machine equipped with plate-type clamps. The loading rate (and also the unloading rate in the case of cycling tests) was maintained at 0.1 inch/minute for the in-machine direction and 0.2 inch/minute for the cross-machine direction. An auto-graphically recorded load-elongation curve was obtained for each test specimen. Tensile work data were obtained from the curves by means of a planimeter.

When the load reached zero during an unloading phase of a test involving cycling, the crossheads of the testing machine were permitted to move toward each other (at the prescribed rate) for approximately five seconds.

Thereafter, the crosshead motion was reversed and they were moved apart at the given rate. Approximately five seconds after the reversal of crosshead motion the specimen again picked up load and the succeeding loading phase was begun. Thus, the time interval between load cycles was on the order of ten seconds, irrespective of the number of the cycle. This procedure is in contrast to an alternate method where the crossheads may be permitted to return to their rest position between cycles; in this latter method the recovery period between cycles increases progressively from cycle to cycle.

The following sections describe details of the test procedure which were specific to each of the three types of tension tests performed in this study.

VIRGIN TENSILE TEST

A conventional virgin tensile test was performed on a sample of ten specimens of each material in each principal direction at the beginning and again at the end of the test program.

ONE-CYCLE TENSILE TEST

The purpose of the one-cycle tensile test was to determine the reload characteristics (slope $\underline{S_r}$) of each sample of sack paper by means of an abbreviated test program. Although the reload characteristics can be determined with somewhat greater accuracy by means of a many-cycle test, it is believed that a one-cycle test procedure will approximate the reload characteristics sufficiently well to be justified from the standpoint of practical application of the fatigue life theory.

The one-cycle test was performed by loading a given specimen to approximately 95% of the average virgin tensile strength of the sack paper, as determined by the preceding virgin tensile test. Thereupon, the specimen was unloaded and then reloaded to rupture. The 95% load levels were 30, 18, 30 and 15 lb./in. for the R-in, R-cross, T-in and T-cross samples, respectively. Because of variability in the tensile strength of individual specimens within a sample, one or two of the ten specimens usually ruptured before the 95% load level was reached. These specimens were replaced by extras so that a total of ten successful tests were performed for each sample.

FATIGUE LIFE TESTS

Fatigue life tests were performed by repeatedly subjecting a specimen to a constant applied strain. That is, in each cycle the applied elongation, measured from the inception of load build-up to the point of load reversal, was maintained constant. In general, four levels of applied strain were prescribed for each sample of sack paper and each direction of test, as listed in Table II. In each case, one of the applied strain levels was at the theoretical endurance limit which, it may be recalled, is the level of repetitively applied strain for which the fatigue life is theoretically infinite. In the cases of Run R, machine direction, and Run T, cross-direction, the least applied strain was selected below the endurance limit as a further check on the accuracy of this theoretical property. It may be noted from Table II that the levels of applied strain for the machine-direction tests ranged from about 50 to 80% of the virgin stretch and from about 25 to 55% of the virgin stretch for the cross-machine tests. The precision of repetitively applying a constant level of strain was $\pm 3\frac{1}{2}\%$ or better.

In the interest of economy of testing time it was decided to terminate the test after a specimen had sustained twenty safe cycles. The selection of twenty cycles as the cut-off point of the fatigue test reflects the interest of the current study in fatigue lives of the same order of magnitude as the fatigue life of a sack in the constant height face drop test. If a specimen reached the cut-off point, it was thereafter loaded to rupture to give an indication of the residual stretch which remained in it.

TABLE II
MAGNITUDE OF APPLIED STRAIN IN FATIGUE LIFE TESTS

Sample	Direction	Trial	Applied Strain, %	Virgin Stretch, %	Ratio, Virgin Stretch	Applied Strain
R	In	1	1.30	1.83		0.71
		2	1.08			0.59
		3	1.01			0.55
		4	0.90			0.49
R	Cross	1	1.46	2.57		0.57
		2	1.34			0.52
		3	1.21			0.47
		4	1.09			0.42
T	In	1	1.07	1.33		0.80
		2	0.93			0.70
		3	0.90			0.68
T	Cross	1	2.46	5.49		0.45
		2	1.71			0.31
		3	1.47			0.27
		4	1.33			0.24

CALCULATION OF THEORETICAL FATIGUE LIFE

The equation which has been derived for the theoretical uniaxial fatigue life of regular sack paper (constant applied elongation) is (3):

$$N = \frac{\log \left\{ \frac{(e_a/e_v) - \alpha - (\beta - \alpha)(e_o/e_v)}{(1 - \alpha)(e_a/e_v) - (\beta - \alpha)(e_o/e_v)} \right\}}{\log (1 - \alpha)} \quad (1)$$

where N = fatigue life

e_a = applied elongation

e_v = virgin elongation

e_o = elongation at limit of proportionality

$\alpha = S_p/S_r$

$\beta = S_o/S_r$

S_o = initial slope of virgin load-elongation curve

S_p = plastic slope of virgin load-elongation curve

S_r = slope of average reload curve in repeated tension.

It may be seen from Equation (1) that the theoretical fatigue life is dependent on (a) one parameter descriptive of the repeated tension process, namely, applied elongation, e_a , and (b) five load elongation parameters which characterize the behavior of the paper in repeated tension, namely, e_v , e_o , S_o , S_p , and S_r . Four of these material parameters (the exclusion being virgin stretch, e_v) may be defined in terms of straight-line approximations to the virgin load-elongation curve and the reload curves, as illustrated in Fig. 1.

It may be anticipated that the accuracy of the fatigue life equation is dependent on (a) the adequacy of the underlying concept of the mechanics of

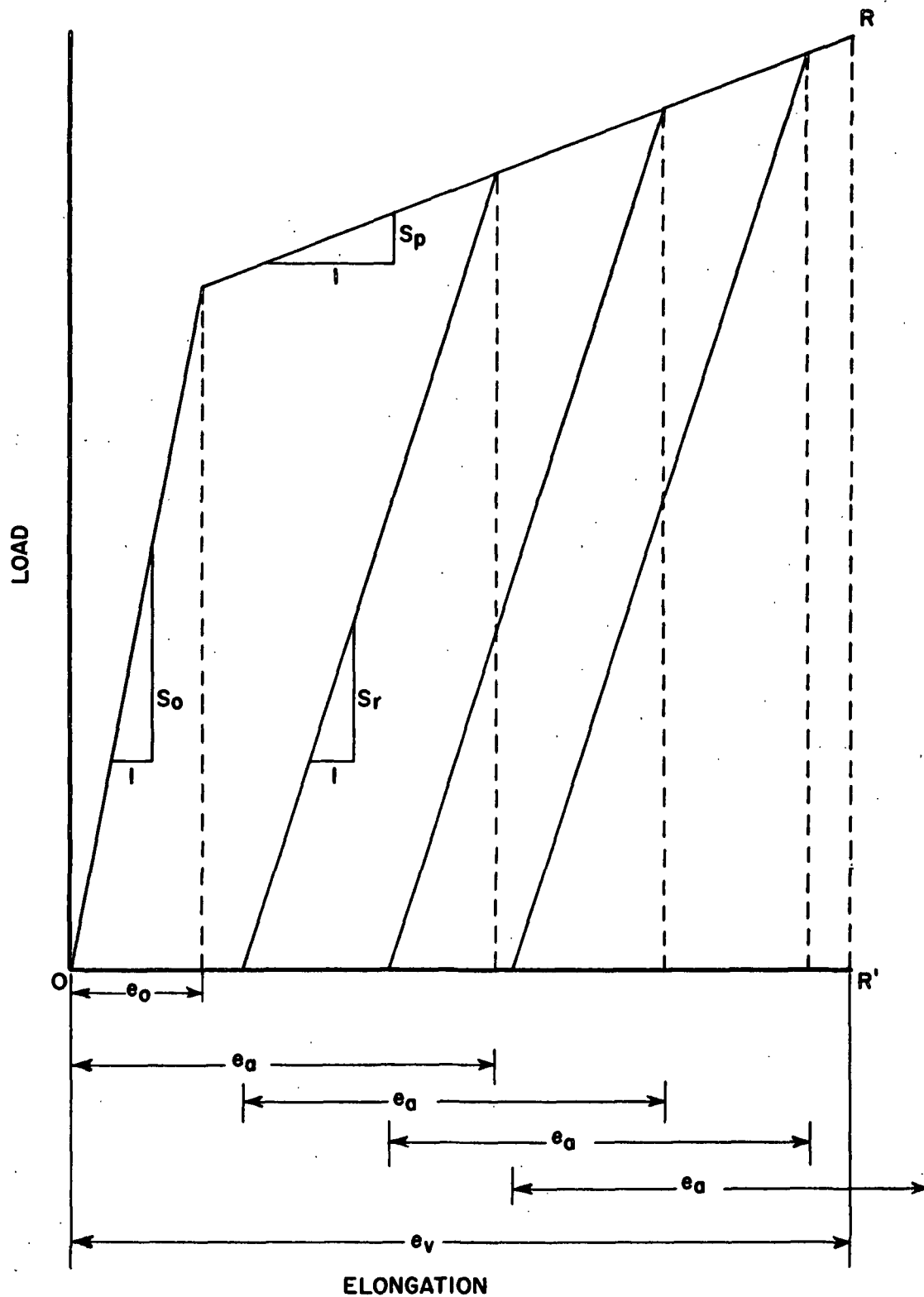


Figure 1. Load-Elongation Properties Employed in Fatigue Theory

repeated tension and (b) the precision of the straight-line approximation to the load-elongation curve. It is pertinent at this point, therefore, to give detailed consideration to the methods of approximating the virgin and reload curves by straight lines.

APPROXIMATION TO THE VIRGIN LOAD-ELONGATION CURVE

A few remarks may be appropriate regarding the objectives which should be kept in mind when approximating the virgin load-elongation curve. A stress-strain (or load-elongation) curve is a continuous collection of points which describe the strain corresponding to a given stress and vice versa. Although it may not always be obvious from an inspection of the results, most analyses of the mechanics of deformable solids at some stage require a mathematical relationship between stress and strain. For example, the bending stress equation met in elementary strength of materials, $\sigma = \frac{M c}{I}$, (σ = stress, M = bending moment, I/c = section modulus) utilizes in its development the approximation that stress, σ , and strain, ϵ , are related by a linear equation, viz., $\sigma = E \epsilon$, where E is the modulus of elasticity (7). Examination of the derivation of the fatigue life equation (3) reveals that at several stages a mathematical relationship between load and elongation was used.

Recognizing the need for a mathematical relationship between stress and strain, it may now be considered what attributes the relationship should have. While mathematical tools exist for approximating an experimentally determined curve such as a load-elongation curve to any desired degree of accuracy, such approximations may become so complicated and unwieldy that the

subsequent mechanical analysis could not be easily carried out. Thus, practical considerations may require some compromise between rigor and simplicity in the approximating equation, recognizing that simplicity may entail a sacrifice of accuracy. It was this consideration of practicality which suggested that the load-elongation curves in repeated tension be approximated by straight lines, inasmuch as linear relationships seldom offer any great difficulty in analytical work.

Lastly, it may be appreciated that the approximating equation should fit the load-elongation curve (as well as possible) over the range of loads and elongations which are of interest to the problem at hand. To cite the beam stress equation again, many metallic structures are designed so that stresses will not exceed the elastic range. Thus, the linear relationship between stress and strain which is inherent in the beam stress equation is entirely adequate in view of the fact that most metals have a linear (or almost linear) stress-strain curve in the elastic range. If, however, the beam structure were to be designed to withstand inelastic stresses, the elementary beam equation would no longer be adequate. Derivation of an analogous equation requires that a suitable approximation be employed covering the inelastic (curvilinear) portion of the stress-strain curve up to the maximum stress of interest (8). In the case of the sack paper fatigue life analysis, all loads and elongations from zero to rupture are involved; thus, the errors incurred in the approximation to the load-elongation curve should be more or less impartially distributed (i.e., averaged out) over the entirety of the load-elongation curve if the straight lines are drawn appropriately.

In summary, the approximation to the load-elongation curve which is required in the fatigue life analysis should be (a) as simple as possible and (b) as good an approximation as possible for the entire load-elongation curve.

Most tensile load-elongation curves for regular sack paper are more or less curvilinear throughout their entirety. Seldom, if ever, can two straight lines be drawn which will exactly fit the entire load-elongation curve. The cross-direction tensile curve for a regular sack paper having high stretch in that direction will frequently come closest to being two straight lines, although even then there may be considerable error in the approximation in the vicinity of the "knee" of the load-elongation curve.

In the interest of establishing a systematic and rapid procedure for fitting two straight lines to a load-elongation curve, a graphical method was devised based on the following considerations. Figure 2 is a tracing of a virgin load-elongation curve in the machine direction from this investigation (Run R). If a tangent is drawn to the curve at the origin, O , and also at the point of rupture, R , the tangent lines will intersect at some point A . From A a line \overline{AB} is constructed which is perpendicular to the load-elongation curve at point B . This construction may be accomplished readily with reasonable accuracy by means of (a) a transparent straight edge, or (b) the common graphical technique of a mirror standing on edge (9). Then the chords \overline{OB} and \overline{RB} are constructed.

Considering point B as dividing the load-elongation curve into two portions, it is apparent that the best straight-line approximation to the curve OB is a straight-line segment passing through point O lying between line \overline{OB}

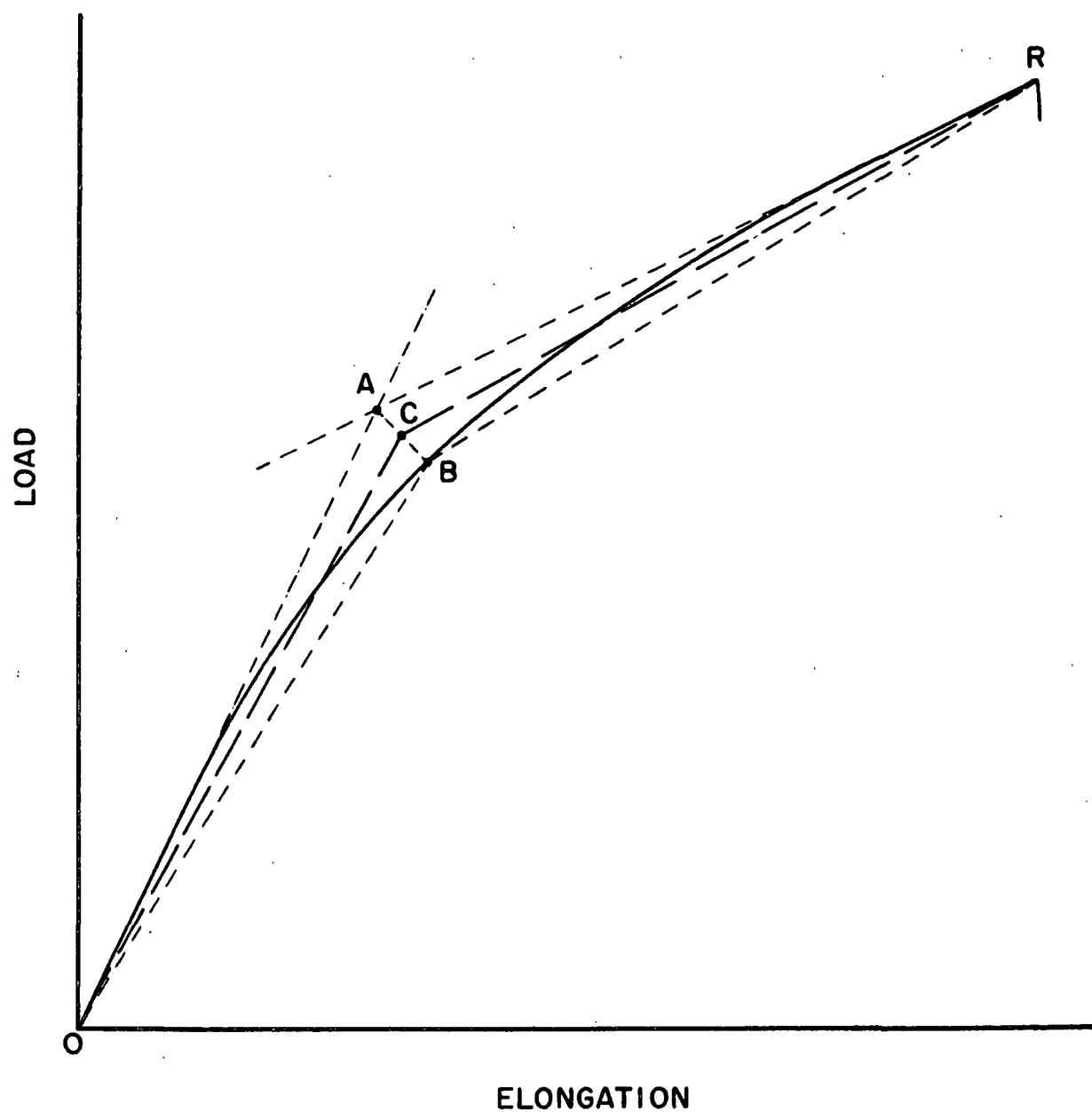


Figure 2. Graphical Constructions Involved in Formulating a Systematic
Method of Approximating Virgin Load-Elongation Curve

and line \overline{OA} . Similarly, the straight line of best fit for the second portion of the load-elongation curve is a segment passing through R and lying between \overline{RA} and \overline{RB} . Inspection of the virgin tensile curves of this study, which represented both average and extreme shapes, suggested that a reasonable approximation to all these diverse shapes could be achieved by having one line segment \overline{OC} bisect the angle between \overline{OA} and \overline{OB} and the second segment \overline{RC} bisect the angle between \overline{RA} and \overline{RB} . An equivalent method of locating \overline{OC} and \overline{RC} is to construct point C as the bisector of the perpendicular segment \overline{AB} ; this may be easily done with a scale or, more elaborately, with compasses.

In practice, the graphical constructions described above may be abbreviated as follows: (See Fig. 3).

(1) Construct tangent line \overline{OA} at the origin of the load-elongation curve and tangent line \overline{RA} at the rupture point.

(2) By means of a transparent scale, estimate the perpendicular line segment from point A to the curve and thereafter establish point C by taking half of \overline{AB} .

Application of the approximation in the fatigue life equation requires the slopes of the two approximating lines (\overline{OC} and \overline{RC}) rather than their formal equations. The slopes may be readily calculated by the following additional steps, which do not demand that the approximating lines be constructed graphically:

(3) Read load \underline{P}_0 and elongation \underline{e}_0 corresponding to point C.
(Elongation \underline{e}_0 is a parameter also required in evaluating the fatigue life equation.)

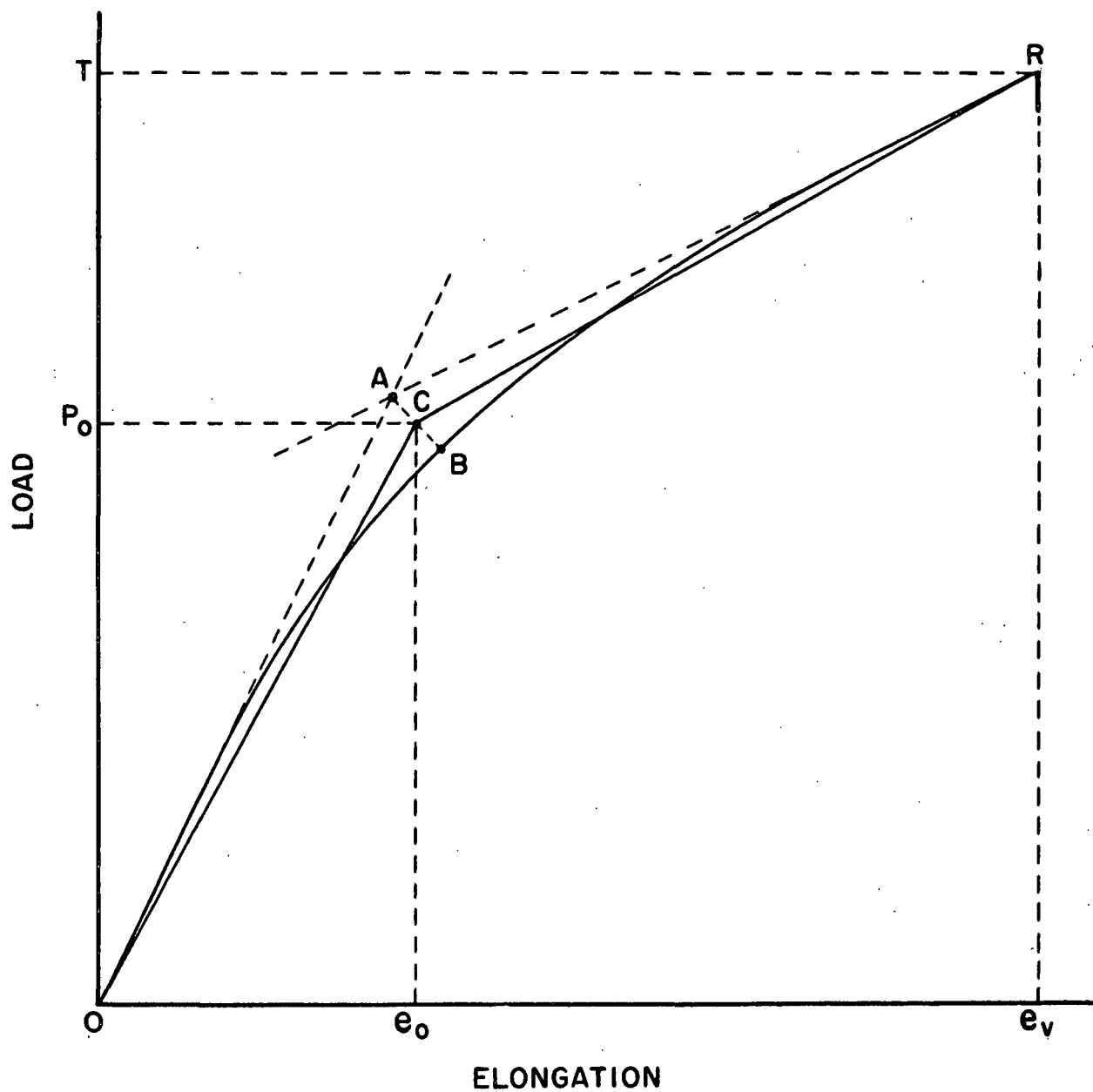


Figure 3. Practical Method of Constructing Straight-Line Approximations
to Virgin Load-Elongation Curve

(4) Read tensile strength, T , and virgin stretch, e_v . (Stretch, e_v , is also required in evaluating the fatigue life equation.)

(5) Calculate the initial slope, S_o , and plastic slope, S_p , from the data of Steps (3) and (4) by means of the following equations:

$$S_o = P_o/e_o \quad (2)$$

$$S_p = (T - P_o)/(e_v - e_o) \quad (3)$$

It is likely that improvements in the approximation could be achieved for various classes of curve shapes by establishing point C of Fig. 2 and 3 at some optimum point for each class. With the curves of this study, for example, it appeared that C profitably might have been taken nearer to point A for the extremely flat-topped cross-machine curves of Run T. However, for the sake of uniformity and in view of the limited experience at this time, it was decided to locate point C as the bisector of line \overline{AB} for all samples.

APPROXIMATION TO THE RELOAD CURVES

The fatigue life equation is based on approximation to the reload curve of repeated tension by means of a single straight line. Again, inspection of a number of reload curves reveals that the curve is generally not a straight line and, therefore, attention should be given to an expedient method of approximation. Two additional considerations are pertinent, beyond those already discussed for the virgin curve approximation.

The first consideration is concerned with the nature of the error which may be tolerated in approximating the reload curve. It may be recalled that in repeated tension a reload curve usually does not pass exactly through

the point of load reversal of the previous cycle. This discrepancy was ignored in the model of repeated tension used in Reference (3), so that a particular repeated tension curve might appear as illustrated by the solid line curves of Fig. 4. (The unloading phase of each cycle is omitted in Fig. 4.)

A study of the derivation of the fatigue life equation (3) will reveal that the analysis is directly concerned with the pairs of points B and A, E and D, G and F, etc., (see Fig. 4) and the straight line joining each pair of points. This is the case because expression of the increments of nonrecoverable stretch (which is the crux of the analysis) involves the elongations and loads at the above-mentioned points. The path of the reload curve from B to A (and from E to D, etc.) is not really of paramount importance to the analysis; rather, the slopes of the dashed-line chords \overline{BA} , \overline{ED} , etc., suffice to relate the pertinent loads and elongations. If, for that particular repeated tension program, one knew all of the chord slopes, they would be sufficient for determining the increments of nonrecoverable stretch and hence for predicting the fatigue life.

Thus, the first consideration to be kept in mind in approximating the reload curves is that it is appropriate to be concerned with the slope of the chords joining the point of load pickup and the point of load reversal of the previous cycle. This is in contrast to the approximation of the virgin curve where it was desired to "average out" the error of the approximation.

Secondly, it may be recalled that the reload curves of regular sack paper in repeated tension not only are generally of lesser slope than the initial slope but also frequently are not parallel among themselves.

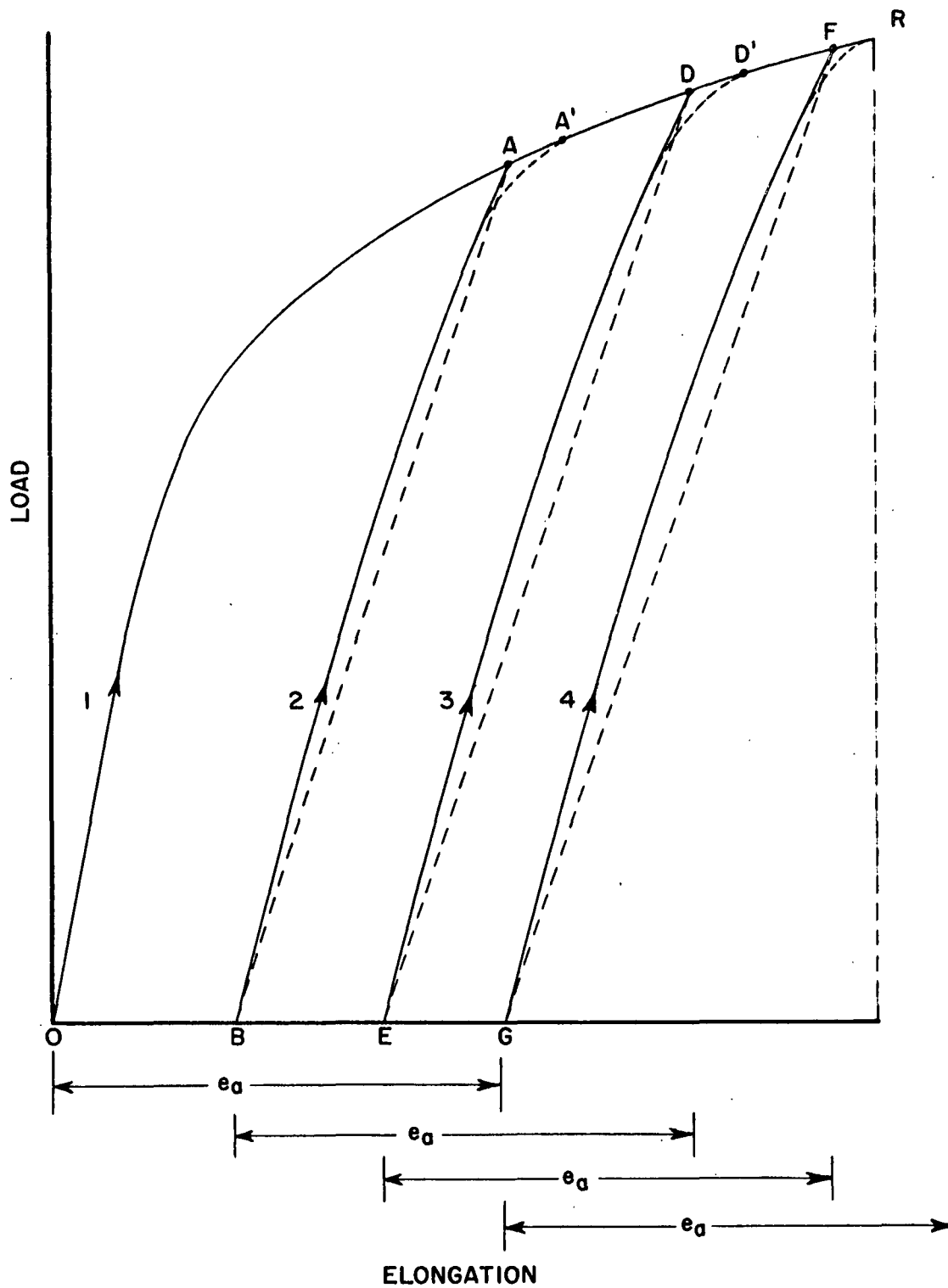


Figure 4. Typical Idealized Repeated Tension Curve Showing Chord
Approximation to Reload Curves

Particularly in the machine direction, successive reload slopes exhibit a progressive clockwise rotation. Inasmuch as it would have hopelessly complicated the analysis to try to account for these differing reload slopes, it was decided to approximate all of them by a single average reload curve.

There are, of course, numerous ways in which an averaging process can be accomplished, depending for the most part on how many reload curves are considered for a given repeated tension test. One fairly direct means of determining an average reload curve is to cycle a specimen to two different load levels--one load level slightly above the "proportional limit" and the second load level near tensile rupture--as illustrated in Fig. 5. Denoting the first and second reload slopes (i.e., slopes of the chords) by \underline{S}_1 and \underline{S}_f , respectively, an estimate of the average reload slope, \underline{S}_r , is given by

$$S_r = (S_1 + S_f)/2. \quad (4)$$

For the samples of this investigation the points of load reversal for the two cycles were arbitrarily selected to be approximately 65 and 95% of the virgin tensile strength. The 65% point in all cases was moderately into the nonlinear portion of the curve. It was found, however, that an estimate of \underline{S}_r differing only slightly from that given by Equation (4) could be obtained by averaging \underline{S}_f (see Fig. 5) and \underline{S}_0 , the initial slope of the virgin curve; that is,

$$S_r = (S_0 + S_f)/2. \quad (5)$$

On the average, the latter estimate was only 5% greater than the former. The estimate given by Equation (5) requires that only a one-cycle rather than a

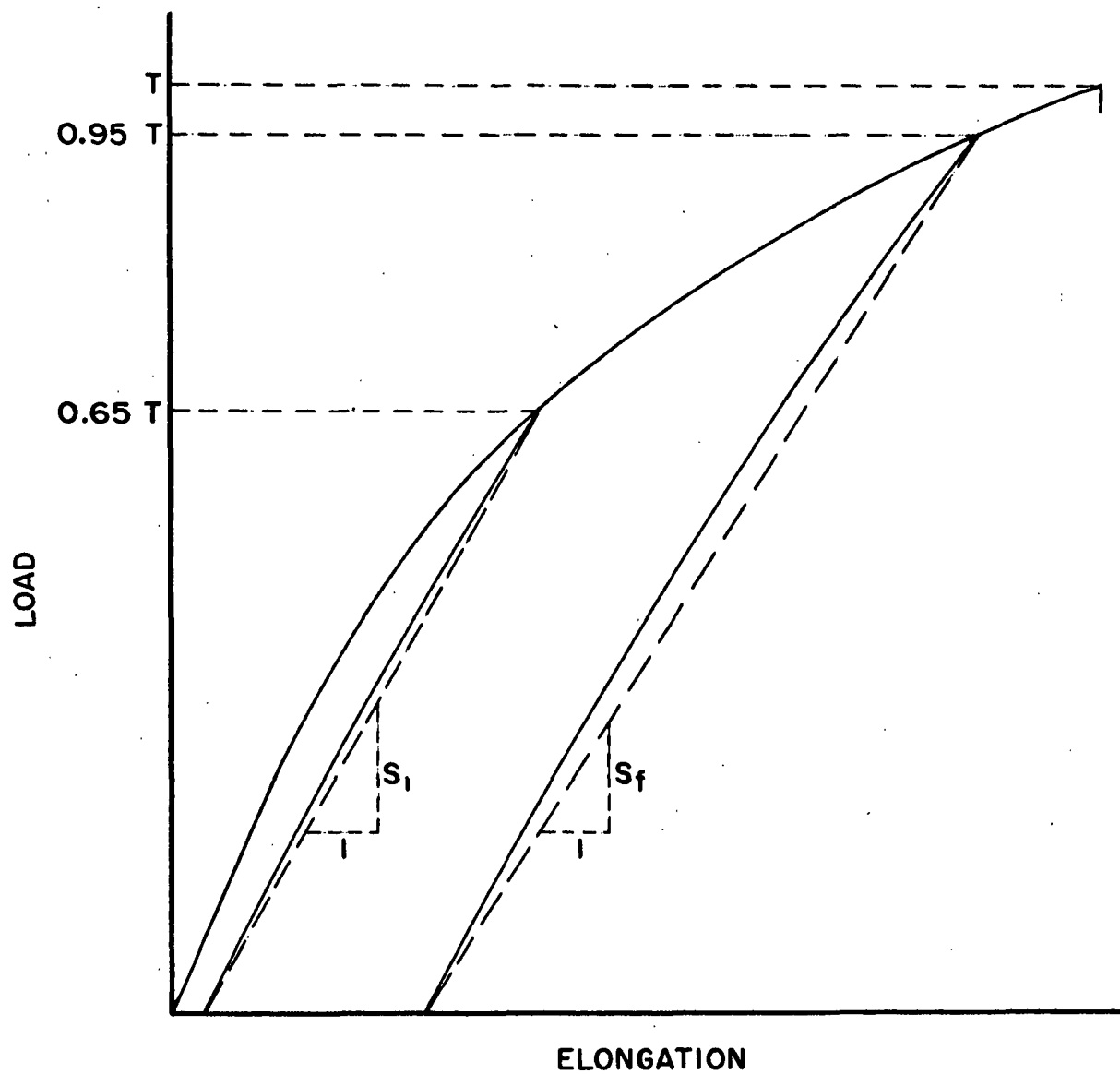


Figure 5. Determination of Reload Slopes by Cycling to 65 and 95% of Virgin
Tensile Strength

two-cycle test be performed on each specimen, and is therefore a saving in test time. Moreover, the value of \underline{S}_r from Equation (5) has the effect of decreasing the estimate of fatigue life somewhat; that is, conservative estimates of fatigue life are obtained and this may be looked upon as a good practice.

In summary, an expedient method of determining an estimate of the average reload slope, \underline{S}_r , is:

(a) perform a one-cycle test to a load level about 95% of the virgin tensile strength, as illustrated in Fig. 6.

(b) find the slope, \underline{S}_f , of the chord joining the point of load pickup on the second loading and the point of load reversal on the first loading.

(c) Obtain \underline{S}_r by averaging \underline{S}_f and the initial slope, \underline{S}_0 , of the virgin tensile curve.

EXAMPLE OF CALCULATION OF A THEORETICAL FATIGUE LIFE

As an example of the calculation of fatigue life from Equation (1), consider the case of sack paper from Run R subjected to repeated constant applied elongation $\underline{e}_a = 0.080$ in. (52% of virgin stretch) in the cross direction.

(1) Following the procedure outlined above for approximating the virgin load-elongation curves, the following average properties were obtained:

Initial slope = $\underline{S}_0 = 350$ lb./sq. in.

Plastic slope = $\underline{S}_p = 54$ lb./sq. in.

Proportional limited elongation = $\underline{e}_0 = 0.037$ in.

Virgin elongation = $\underline{e}_v = 0.154$ in. (i.e., 2.57% stretch)

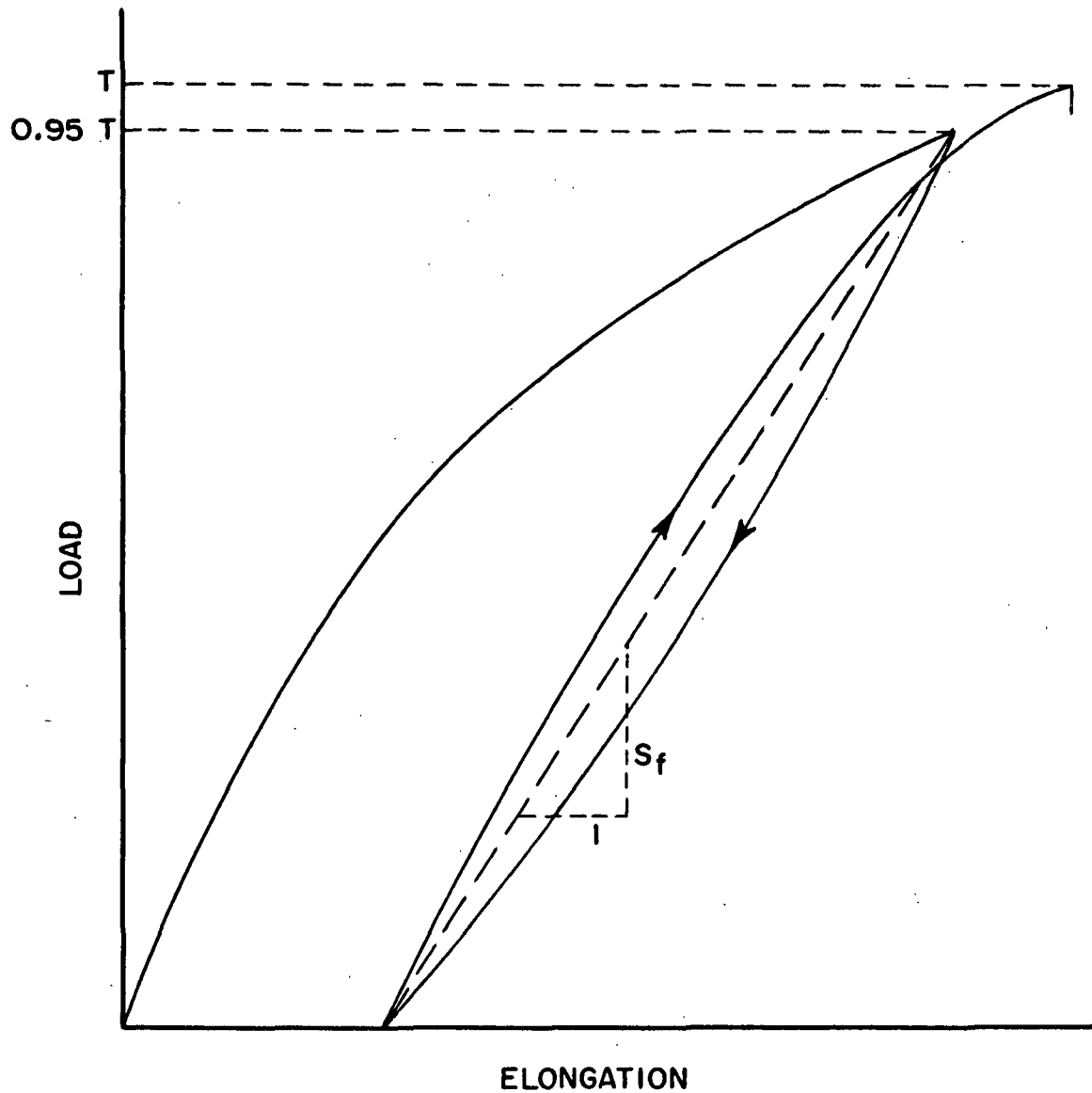


Figure 6. Single-Cycle Tension Test for Determination of Reload Slope

(Although the initial slope, \underline{S}_0 , has dimensions of pounds per square inch, it should not be confused with the modulus of elasticity, \underline{E} , which has the same dimensions. The slope \underline{S}_0 was taken from an experimental curve of tension load on a one-inch wide strip vs. elongation [rather than from a curve of stress vs. strain] and therefore differs from the modulus of elasticity by a factor of [caliper/span]. \underline{S}_0 is about three orders of magnitude lower than \underline{E} .)

(2) Following the procedure described earlier for analyzing the single-cycle test, the reload slope at the point corresponding to 95% of the tensile strength was found to be $\underline{S}_f = 243$ lb./sq. in. The average reload slope therefore is estimated to be

$$\underline{S}_r = (\underline{S}_0 + \underline{S}_f)/2 = (350 + 243)/2 = 296 \text{ lb./sq. in.}$$

(3) From the data of Steps (1) and (2) the dimensionless parameters required in the fatigue life equation are:

$$e_a/e_v = 0.080/0.154 = 0.52$$

$$e_o/e_v = 0.037/0.154 = 0.24$$

$$\alpha = \underline{S}_p/\underline{S}_r = 54/296 = 0.18$$

$$\beta = \underline{S}_0/\underline{S}_r = 350/296 = 1.18$$

(4) Substituting in Equation (1):

$$N = \frac{\log \left\{ \frac{(e_a/e_v) - \alpha - (\beta - \alpha)(e_o/e_v)}{(1 - \alpha)(e_a/e_v) - (\beta - \alpha)(e_o/e_v)} \right\}}{\log (1 - \alpha)}$$

$$= \frac{\log \left\{ \frac{0.52 - 0.18 - (1.18 - 0.18)(0.24)}{(1 - 0.18)(0.52) - (1.18 - 0.18)(0.24)} \right\}}{\log (1 - 0.18)}$$

$$\begin{aligned} &= \frac{\log \left(\frac{0.100}{0.186} \right)}{\log (0.82)} = \frac{\log (0.538)}{\log (0.82)} \\ &= \frac{\log (5.38 \times 10^{-1})}{\log (8.20 \times 10^{-1})} = \frac{0.7308 - 1.0000}{0.9138 - 1.0000} \\ &= \frac{-0.2692}{-0.0862} = 3.12. \end{aligned}$$

Rounding to the next higher integer as required by the theory, the fatigue life is $\underline{N} = 4$, meaning that the paper can be expected to sustain four cycles safely at this level of applied elongation but should fail on the fifth application of elongation.

DISCUSSION OF RESULTS

EXPERIMENTAL FATIGUE LIFE

Repeated tensile tests were performed at various levels of applied strain on two samples of regular 50-lb. sack paper in each principal direction for the purpose of determining their uniaxial fatigue lives. The levels of applied strain were selected for each sample and direction so as to afford a comparison with the theoretical fatigue lives at both high and low magnitudes.

The data on experimental fatigue life are presented in Table III. The fatigue life of each individual specimen is presented; the specimens within a sample are arranged in the order of increasing fatigue life rather than in the order of testing.

With reference to Table III, the ten machine-direction specimens of Sample R exhibited fatigue lives of generally two and in one case three when subjected to a repeated applied strain $\underline{e_a} = 1.30\%$ (about 71% of the virgin stretch). A fatigue life of two means that the specimen safely withstood two cycles of applied strain but failed on the third application before the contemplated level (1.30%) was reached. A fatigue life of three means that the specimens sustained three cycles but failed during the fourth application. (The virgin tensile test, where of course the applied strain equals the virgin stretch, corresponds to a fatigue life of zero--no cycles sustained before failure.)

When a level of strain $\underline{e_a} = 1.08\%$ was applied to machine-direction specimens from Run R, they exhibited fatigue lives ranging from three to eleven as shown in Table III. The next lower level of applied strain ($\underline{e_a} = 1.01\%$) gave

TABLE III

OBSERVED FATIGUE LIFE AT VARIOUS LEVELS OF APPLIED STRAIN

Sample	Direction	Specimen Number	Fatigue Life Applied Strain, ea, %			
			1.30	1.08	1.01	0.90
R	In	1	2	3	6	9
		2	2	4	10	14
		3	2	5	11	18
		4	2	5	14	18
		5	2	6	15	> 20
		6	2	6	17	> 20
		7	2	8	19	> 20
		8	2	9	> 20	> 20
		9	2	9	> 20	> 20
		10	3	11	> 20	> 20
		Mean	2.1	6.6	> 15.2	> 17.9
		Median	2	6	16	> 20
Sample	Direction	Specimen Number	Fatigue Life Applied Strain, ea, %			
			1.46	1.34	1.21	1.09
R	Cross	1	2	3	7	4
		2	3	3	8	5
		3	3	3	10	9
		4	3	4	12	11
		5	3	5	13	13
		6	3	5	13	16
		7	3	6	16	16
		8	4	6	18	> 20
		9	5	7	> 20	> 20
		10	5	9	> 20	> 20
		Mean	3.4	5.1	> 13.6	> 20
		Median	3	5	13	> 20
Sample	Direction	Specimen Number	Fatigue Life Applied Strain, ea, %			
			1.07	0.93	0.90	
T	In	1	1	2	0	1
		2	1	2	1	2
		3	1	2	1	2
		4	1	2	1	3
		5	1	2	3	6
		6	1	2	3	7
		7	1	3	8	8
		8	2	3	8	11
		9	2	6	8	12
		10	2	7	12	12
		Mean	1.3	3.8	> 6.0	
		Median	1	2.5	5	
Sample	Direction	Specimen Number	Fatigue Life Applied Strain, ea, %			
			2.46	1.71	1.47	1.33
T	Cross	1	3	6	11	9
		2	3	9	18	9
		3	3	9	> 20	> 20
		4	3	10	> 20	> 20
		5	4	11	> 20	> 20
		6	4	11	> 20	> 20
		7	4	13	> 20	> 20
		8	4	17	> 20	> 20
		9	5	> 20	> 20	> 20
		10	6	> 20	> 20	> 20
		Mean	3.9	> 12.6	> 18.9	> 17.8
		Median	4	11	> 20	> 20

substantially higher fatigue lives for this material. Three of the ten specimens reached 20 cycles, which was the arbitrary cut-off point of the testing. Fatigue lives for these specimen are listed as ">20," indicating that the actual fatigue life of the specimen was somewhat greater than twenty cycles. As mentioned previously, twenty cycles was selected as the cut-off point because it corresponds to the typical maximum number of impacts sustained by a filled sack in the sack impact test. The fourth and least value of applied strain ($\epsilon_a = 0.90\%$) resulted in an even greater number of specimens reaching the cut-off value, namely, six specimens of ten.

The arithmetic mean and the median are also tabulated in Table III for each sample. A median, it may be recalled, is the value for which half of the number of specimens in the sample exhibit greater values and half exhibit lesser values. The median fatigue life of a sample is frequently favored in the analysis of fatigue data because it is influenced less than the arithmetic mean by extreme values (10). Inasmuch as the distribution of fatigue life is often skewed in the direction of higher fatigue lives, the median is sometimes regarded as the more meaningful estimate of the "average" fatigue life. It may be noted in Table III that in nine out of fifteen cases the median is somewhat less than the mean. For those samples where one or more specimens reached the 20-cycle cut-off point, the mean (and in some instances the median also) must be tabulated as "greater than" the given numerical value.

One of the more noteworthy aspects of the data of Table III is the large variation in tensile fatigue life within a sample at the lower values of applied strain. In three cases the variability seemed so inordinately high upon first inspection that repeat trials were performed on additional samples

of size ten, as indicated in Table III. The most extreme variability was with Run T in the machine direction. The fatigue lives at the lowest applied strain ranged from one cycle to greater than 20 cycles--a twentyfold range. At the intermediate level of strain, the specimen lives of Run T ranged from zero to twelve--the zero life meaning that the specimen did not survive the first application of strain even though in this instance it was only 70% of the virgin stretch.

High variability in fatigue life seems to be characteristic of paper, as noted by Mappus (6), and of materials in general (11). In view of the notoriously high variability in the virgin stretch of paper, it perhaps should not be too surprising that repeated tension is even more variable inasmuch as there may also be expected to be comparable variability in its other pertinent properties (i.e., its recoverable and nonrecoverable stretch characteristics). Moreover, it is not unreasonable that the variability becomes more severe at lower levels of applied strain; then, differences which exist between specimens with regard to virgin stretch and recoverable stretch become relatively larger compared to the applied elongation.

The variability exhibited by these samples, however, is no worse than the variability exhibited by the drop test performance of sacks made therefrom. When the constant height, face drop data of Table V from Reference (1) are expressed in terms of fatigue life, it is found that the sack fatigue lives of Run R ranged from 0 to 28 and from 4 to 39 for Run T. Sack fatigue undoubtedly reflects conversion variability in addition to paper variability.

Regarding variability in sack paper fatigue life, it may be appreciated that a difference of one unit in fatigue life is probably not experimentally significant. The reason is that fatigue life is by definition a discrete variable, being always an integer. That is, fatigue life may be 5, for example, or 6, but never an intermediate value. Two specimens with virtually identical residual stretch at a late stage in a repeated tension test could easily exhibit fatigue lives differing by one unit for the following reason. An application of elongation may be just sufficient to exceed the residual stretch of one specimen but not quite enough to exceed the other. The higher stretch specimen would survive one more cycle.

The inevitability of differences of at least one unit of fatigue life between specimens within a sample may be strikingly demonstrated by the following consideration. Suppose ten specimens from a given sample are subjected to an applied elongation equal to the average virgin stretch of the paper, as determined by a prior virgin tensile test. Ostensibly the test under consideration is a virgin tensile test because the applied elongation is just equal to the virgin stretch and the fatigue life should be zero. Because of variability in the sample, however, about five of the specimens can be expected to have a stretch less than the average virgin stretch; these specimens will fail under the first applied elongation, i.e., zero fatigue life. The remaining five or so specimens, on the other hand, will have stretch greater than the average and they will not fail under one application; rather they will sustain at least one safe application, i.e., they will exhibit a fatigue life of at least unity. Thus, differences of at least one unit in fatigue life within a group of specimens are to be expected. For practical purpose, therefore,

specimens whose fatigue lives are one unit different can perhaps be considered as being identical. Stated differently, one perhaps should not attach any great significance to differences of one unit in fatigue life tests. Samples R and T at their highest levels of applied elongation in the machine direction are cases in point. See Table III. The median fatigue lives of two and one, respectively, are probably realistic values for these samples and from a practical standpoint there is no real variability in fatigue life within each sample.

A graph of median fatigue life vs. applied strain is presented in Fig. 7, with the coordinate axes arranged in the conventional sense of the S-N diagrams of classical fatigue analysis. With two of the curves the highest experimental values of fatigue life were cut-off values at twenty; the actual fatigue life is somewhat greater. This fact has been taken into consideration in visually fitting a curve to the plotted points. The virgin tensile test (zero fatigue life) has also been included as an experimental point in Fig. 7 except for Run T-CD (stretch = $5-1/2\%$) where inclusion would have hampered the scale. Although the virgin test is not conducted in strictly the same manner as the fatigue test (if it were, the median life might be either zero or unity, as discussed above), the virgin test nonetheless represents an end-point of the fatigue life curve.

The curves of Fig. 7 exhibit the general shape anticipated for fatigue life data. It may be noted that the relationship between fatigue life and applied strain is highly nonlinear over the range of zero to twenty cycles. Beyond a life of four or five cycles (or about ten cycles in the case of Run T, cross direction) the fatigue life increases very rapidly with small decreases

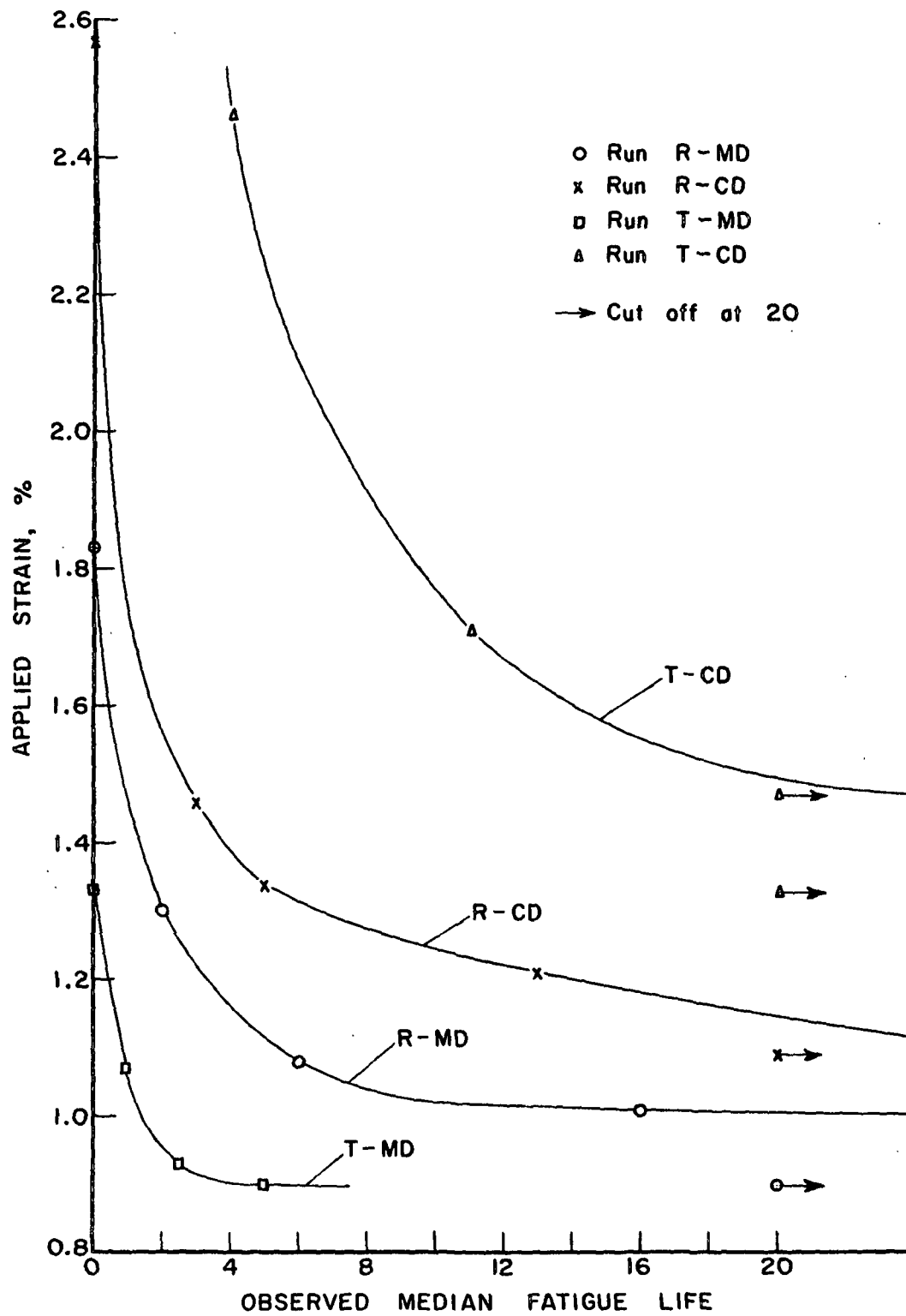


Figure 7. Relationship Between Observed Fatigue Life and Applied Strain
for Two Samples of Regular 50-lb. Kraft Sack Paper

TABLE IV
COMPARISON OF SACK IMPACT PERFORMANCE OF RUNS R AND T

Type of Impact	Safe Inches		Sack Fatigue Life (Mean)	
	Run R	Run T	Run R	Run T
<u>Pasted Sacks</u>				
Progressive height face drop	417	600	8.8	11.1
Constant height face drop	530	933	11.0	19.4
Butt drop	44	77	3.7	6.4
<u>Sewn Sacks</u>				
Progressive height face drop	349	553	7.8	10.5
Constant height face drop	419	898	8.7	18.7
Butt drop	35	68	2.9	5.7

in applied strain. The fact that the curve of Run T, cross direction, is a more gradually "breaking" curve is attributable to its high degree of plasticity, as discussed in theoretical terms in Reference (3).

Figure 7 indicates that if the same (though arbitrary) strain were applied repetitively to the cross-direction of both Runs R and T, a markedly higher fatigue life would be exhibited by Run T. This test condition is probably analogous to the Frag impact fatigue test, in the sense that the Frag test applies a constant input to all samples (for a given drop height). Table I shows that the Frag falls-to-failure was 89 for Run T and 52 for Run R (cross direction). Thus, both a tensile fatigue test and the Frag test rank the samples in the same order.

Similarly, Fig. 7 indicates that, for the machine direction, Run R has a higher fatigue life than Run T within the range of experimental data. The Frag cross-machine test ranked these samples in the same order, namely, 29 vs. 12 falls-to-failure. Thus, the tensile fatigue life data is compatible in a broad sense with the Frag impact fatigue data for samples of this study.

Table IV lists the sack impact performance of Runs R and T for pasted and sewn sacks in progressive and constant height face drop and butt drop. The data are extracted from Reference (1). Sack fatigue life is tabulated in addition to accumulated safe inches of drop; these two measures of impact performance are related as discussed in Reference (3).

It may be seen that the sack drop performance of Run T was better than that of Run R in all cases. With reference to Fig. 7, the cross-machine tensile fatigue test (at any level of applied elongation) ranks Runs R and T

in the same order as their sack performance. It may be recalled that one of the general conclusions of the fabrication program study on regular sack papers was that the better correlations between paper properties and sack performance involved cross-machine properties of the paper. The tensile fatigue life data of this present study is compatible with that general conclusion.

In citing the compatibility of the tensile fatigue data with Frag and sack performance, however, it must be remembered that only two samples with widely differing properties are being considered and it would be hazardous to generalize from these meager results. Nonetheless, it is perhaps pertinent to note that the tensile fatigue behavior of these samples is not at variance with previous experience in the testing of the sack paper and sacks made therefrom.

COMPARISON OF THEORETICAL AND EXPERIMENTAL FATIGUE LIFE

Evaluation of theoretical fatigue life [Equation (1) of a preceding section of this report] requires determination of certain parameters of the virgin and reload curves in uniaxial tension. A detailed description of the methods employed in calculating these parameters was given in the section entitled "Calculation of Theoretical Fatigue Life." It is worth repeating that, with the exception of virgin stretch, these parameters are defined in terms of straight-line approximations to the virgin and repeated tension load-elongation curves.

The numerical values of the fatigue parameters for Runs R and T in both principal directions are listed in Table V. A comparison of in- and cross-direction properties is afforded by the table although these ratios are

TABLE V
LOAD-ELONGATION CURVE PARAMETERS FOR THEORETICAL EVALUATION OF FATIGUE LIFE

	Run R			Run T		
	In	Cross	Ratio, $\frac{In}{Cross}$	In	Cross	Ratio, $\frac{In}{Cross}$
Initial slope of virgin curve, \underline{S}_0 , lb./sq. in.	574	350	1.6	652	258	2.5
Plastic slope of virgin curve, \underline{S}_p , lb./sq. in.	161	54.1	3.2	232	23.6	10.2
Reload slope from single cycle test, \underline{S}_f , lb./sq. in.	481	243	2.0	541	146	3.7
Average reload slope, $\underline{S}_r = (\underline{S}_0 + \underline{S}_f)/2$, lb./sq. in.	528	296	1.8	596	202	3.0
β , $(=\underline{S}_0/\underline{S}_r)$	1.087	1.182	0.92	1.094	1.277	0.86
α , plasticity factor, $(=\underline{S}_p/\underline{S}_r)$	0.305	0.183	1.7	0.389	0.117	3.3
Proportional limit strain, \underline{e}_0 , %	0.52	0.62	0.84	0.57	0.72	0.79
Virgin stretch, \underline{e}_y , %	1.83	2.57	0.71	1.33	5.49	0.24
Ratio, $\underline{e}_0/\underline{e}_y$	0.314	0.240	1.3	0.425	0.131	3.2

not directly involved in the fatigue-life evaluation. A few observations on the magnitude and range of these parameters may be helpful. It may be seen that the initial slope, \underline{S}_0 , of the machine-direction virgin curve exceeds the cross-machine virgin slope by a factor of about 1.6 and 2.5 for Runs R and T, respectively, which is reminiscent of kraft papers (and paperboards) in general. Again, it should be mentioned that although the initial slope (as well as the other slopes in Table V) has dimensions of pounds per square inch, it is not a modulus of elasticity. These slopes are taken from a load-elongation curve rather than a stress-strain curve and are about 1/1000 the magnitude of moduli of elasticity. One could work, of course, with moduli inasmuch as ratios of slopes are involved in the fatigue life equation, but this would require further data reduction of the load-elongation curve.

The plastic slopes, \underline{S}_p , are seen to be markedly less than the initial slopes, as would be expected. Moreover, the cross-machine plastic slopes are extremely low, being exceeded by factors of three and ten by their machine direction counterparts for Runs R and T, respectively.

The reload slopes, \underline{S}_r , corresponding to a single cycle to 95% of the virgin tensile strength, are seen to be substantially less than the initial slopes of the virgin curves, reflecting thereby the previously mentioned trend to rotation of the reload slope during cycling in tension.

For purposes of this study the average reload slope, \underline{S}_r , was taken as the mean of the initial slope, \underline{S}_0 , and the single-cycle reload slope, \underline{S}_f . Values of \underline{S}_r for the two samples of sack paper are also tabulated in Table V.

The fatigue life equation explicitly requires the ratio β of initial slope, S_0 , and reload slope, S_r . Table V reveals that this ratio varies rather modestly from 1.087 to 1.277. That is, the initial slope exceeds the average reload slope by about 9 to 28% for these sack papers. Recalling that these samples represented quite extreme differences in most respects, the tabulated values of β suggest that this factor may not vary widely for regular sack papers.

Much greater differences are exhibited for the second primary parameter of the fatigue life equation, namely, α , the ratio of plastic slope and re-load slope. This factor varies from 0.117 to 0.389, both extremes occurring with Run T. The ratio α , which has been termed the plasticity factor (3), reflects the degree of "flatness" of the plastic portion of the virgin load-elongation curve relative to the reload slope (and in effect, relative to the initial slope of the virgin curve). The cross direction of Run T is highly plastic ($\alpha = 0.117$); this is the high stretch sample of this study with a virgin stretch of 5-1/2%.

The fatigue life equation also involves the magnitude of the proportional limit strain, e_0 , relative to the virgin stretch, e_v . Values of e_0/e_v varied between 0.131 and 0.425 for the samples of this study. As with α , the least value of e_0/e_v reflects the high degree of plasticity and stretch of Run T in the cross direction.

In general, the data of Table V suggest that α ($= S_p/S_r$) tends to be low for the cross direction of regular sack paper and high for the machine direction. The same may be said for the ratio e_0/e_v . The ratio β ($= S_0/S_r$) displays the reverse trend, although it probably does not vary over a wide range.

The theoretical fatigue lives of Runs R and T were evaluated by substituting the values of α , β , and $\frac{e_0}{e_y}$ from Table V into Equation (1), along with each value of the constant applied elongation, $\frac{e_a}{e_y}$ (relative to the virgin stretch) from Table II. An example of the calculation is given in a preceding section of this report. The resulting theoretical fatigue lives are listed in Table VI. Also shown in this table are the median experimental fatigue lives with their maxima and minima (from Table III). The numeral in parentheses following the maximum value denotes the number of specimens in the sample, expressed as a percentage of the sample size, which reached the cut-off value of 20 cycles. A graph of the theoretical fatigue life of each material and direction of test is given in Fig. 8. The experimental fatigue lives are also plotted in Fig. 8 as isolated points not joined by curves.

As an example of the comparison afforded by Table VI and Fig. 8, attention may be directed to the data for Run R when tested in the machine direction. At an applied strain of 1.30% (which is 71% of the virgin stretch) the predicted fatigue life was two cycles and the observed median fatigue life was also two cycles. At the next lesser value of applied strain, 1.08%, (59% of the virgin stretch) the theoretical life was four cycles and the observed median life was six cycles. It may be noted from Table VI that the lives of the individual specimens of this sample ranged from three to eleven, so that the predicted life is within the range of lives exhibited by the individual specimens.

The third level of applied strain for Run R, machine direction, was the theoretical endurance limit, namely, 1.01% (55% of the virgin stretch). It may be recalled that the endurance limit is that value of applied strain for which

TABLE VI
COMPARISON OF THEORETICAL AND EXPERIMENTAL FATIGUE LIVES

Sample	Direction	Virgin Stretch, $\frac{e_v}{\%}$	Applied Strain, $\frac{e_a}{\%}$	Ratio, $\frac{e_a}{e_v}$	Fatigue Life Theoretical	Fatigue Life Observed	Range of Observed Fatigue Life, Min. Max.
R	In	1.83	1.30	0.71	2	2	2 3
			1.08	0.59	4	6	3 11
			1.01	0.55	∞	16	6 >20(30%) ^a
			0.90	0.49	∞	>20	9 >20(60%)
R	Cross	2.57	1.46	0.57	3	3	2 5
			1.34	0.52	4	5	3 9
			1.21	0.47	6	13	4 >20(25%)
			1.09	0.42	∞	>20	>20(100%)
T	In	1.33	1.07	0.80	2	1	1 2
			0.93	0.70	5	2.5	0 12
			0.90	0.68	∞	5	1 >20(5%)
T	Cross	5.49	2.46	0.45	3	4	3 6
			1.71	0.31	9	11	6 >20(20%)
			1.47	0.27	∞	>20	11 >20(80%)
			1.33	0.24	∞	>20	9 >20(80%)

^a Numeral in parentheses expresses percentage of the sample which reached the cut-off fatigue life.

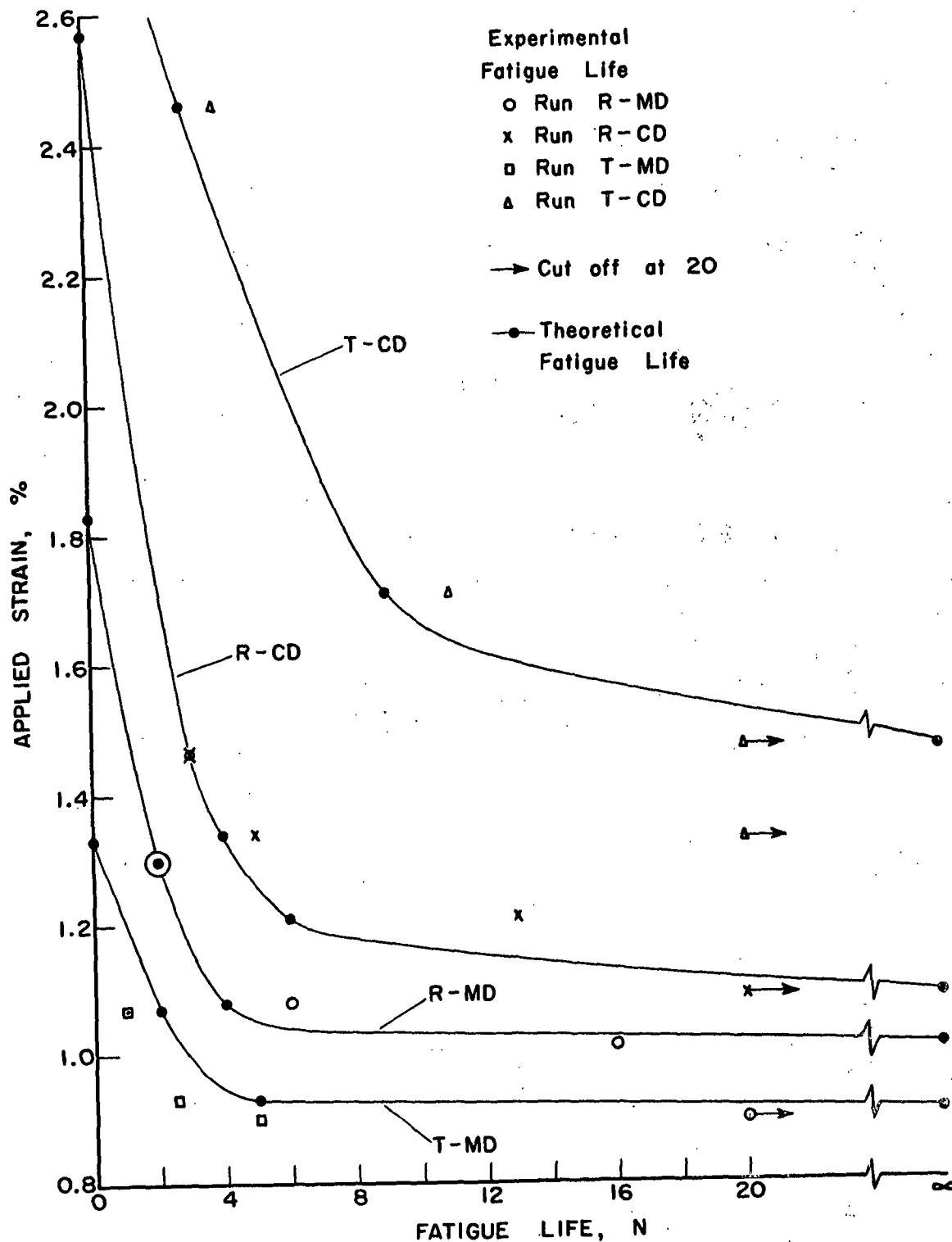


Figure 8. Comparison of Theoretical and Observed Fatigue Lives for Two Samples of Regular 50-lb. Kraft Sack Paper

the fatigue life is theoretically infinite. Although it is doubtful that, practically speaking, any specimen can actually sustain an indefinitely large number of cycles, the concept of an endurance limit is useful in the sense that the fatigue life increases rapidly to large values as the applied strain approaches the endurance limit. That this indeed occurs is suggested by the experimental curves of Fig. 7 for Runs R and T in the machine direction.

Returning to consideration of Fig. 8, the observed fatigue life of Run R at the in-machine theoretical endurance limit was sixteen. Of the ten specimens in the sample, 30% of them (that is, three specimens) attained the cut-off value of twenty, which in this experimental study is the equivalent of an infinite fatigue life. These results suggest that the actual endurance limit for Run R is somewhat lower than the theoretical value of 1.01%.

In assessing the agreement between theory and experiment at the higher levels of fatigue life, it should be recognized that numerically large differences in fatigue life may occur as a result of only very modest differences in applied strain. This happens because the fatigue life curve, as graphed in Fig. 7 or 8, approaches the horizontal at the endurance limit. For example, the trend of the experimental points for Run R, machine direction, suggests that a further reduction in applied strain would lead to a substantial increase in the fatigue life. The fourth level of applied strain in this experiment, namely, 0.90% (49% of virgin stretch) supports this view. In this case the median fatigue life was greater than twenty cycles. Sixty per cent of the specimens in the sample reached the cut-off point. In terms

of this experiment, therefore, it appears that the true endurance limit lies near 0.90% rather than the theoretical endurance limit, 1.01%. These levels correspond to 49 and 55%, respectively, of the virgin stretch.

Turning attention to the cross-machine data for Run R, there appears to be very good agreement between theory and experiment at the two highest levels of applied strain, involving differences of at most one cycle of fatigue life. At an applied strain of 1.21%, however, the agreement is less favorable. While the predicted life was six cycles, the observed median life was 13, with 25% of the specimens reaching the cut-off point. Variability in this sample was particularly high, as shown in Table I-II, with fatigue lives ranging from four cycles to the cut-off.

At the theoretical endurance limit, 1.09% strain, all of the specimens in the sample reached the twenty-cycle cut-off, indicating that the true endurance limit was in the neighborhood of this applied strain.

Taken in their entirety, the data for the cross direction of Sample R suggest that the true fatigue life curve lies somewhat above the theoretical curve for fatigue lives greater than three although the endurance limit is probably near the theoretical value.

The poorest agreement between theory and experiment occurred with the machine-direction specimens of Run T, a paper with low machine direction stretch (1.33%). Theoretically, the fatigue life curve "breaks" very sharply in the vicinity of an applied strain of 0.95%. While the agreement between theory and experiment was fair at the two larger applied strains (2 vs. 1 and 5 vs.

2.5 cycles, respectively), the difference in fatigue lives near the endurance limit was very extreme. The lowest applied strain (0.90%) was just slightly less than the theoretical endurance limit (0.91%); the experimental fatigue life was only five, however, and only one in twenty specimens reached the cut-off point. On the other hand, it may be noted that this sample was extremely variable, with lives ranging from one to >20 cycles. It may be noted from Fig. 8 that the experimental data appears to define a curve which is parallel to the theoretical curve but displaced downward, suggesting a systematic error in the theoretical prediction. Further discussion of this observation will be given in a later section of this report concerning examination of one of the basic concepts on which the fatigue theory is based.

Lastly, with reference to the cross-machine direction of Run T, it may be noted that there was good agreement between theory and experiment. At the two higher levels of applied strain, the predicted fatigue life differed from the observed life by one and two cycles, respectively. When the applied strain was equal to the theoretical endurance limit (1.47%), the median fatigue life was >20 and 80% of the specimens reached the cut-off point. Somewhat surprisingly, in the fourth test when the applied strain (1.33%) was below the theoretical endurance limit, still 80% of the specimens attained the cut-off point; one might have expected all of the specimens would have reached 20 cycles. Except for this aspect, however, the theoretical fatigue life was in generally good agreement with experiment for Run T in the cross-machine direction.

By way of general comment, it may be seen in Fig. 8 that the fatigue life theory appears to account quite well for the extreme nonlinearity between

fatigue life and applied strain. That is, the shape of the theoretical fatigue life curves in all cases seem to be essentially the same as those defined by the experimental data. This observation suggests that the theoretical concepts of repeated tension are in essential agreement with the actual mechanism. In judging individual differences between theory and experiment, it is perhaps appropriate to keep in mind (a) the high variability which seems to be characteristic of fatigue life, and (b) the extreme sensitivity of fatigue life to small differences in applied strain when the latter is near the endurance limit. These two characteristics are probably related to each other.

It may be of interest to compare the magnitude of the endurance limit, as inferred from Fig. 7 or 8, with the proportional limit strain tabulated in Table V. This comparison is facilitated by Table VII.

TABLE VII
COMPARISON OF ENDURANCE LIMIT AND PROPORTIONAL LIMIT STRAIN

Sample	Direction	Proportional Limit, e_p , %	Endurance Limit, ^a e_w , %
R	In	0.52	0.80
R	Cross	0.62	1.00
T	In	0.57	0.80
T	Cross	0.72	1.30

^a Conservative estimate.

It should be recognized that the experimentation was not arranged to determine the endurance limit with great confidence (this would require much larger scale testing). The estimates of endurance limit listed in Table VII, however, are

believed to be conservative (that is, less than the true endurance limit) in view of the course of the remainder of the fatigue life curves in Fig. 7 and 8, and in all cases are less than the theoretical endurance limit.

The proportional limit strains in Table VII are associated with the straight-line approximation to the virgin curve and therefore are actually somewhat greater than the true proportional limits of the paper.

It may be seen in Table VII that in all cases the endurance limit strain is substantially greater than the proportional limit strain. The differences are probably even greater than appear in the data of the table because of the conservatism in the two properties, as discussed above. It may be recalled that the result was predicted by the theory of fatigue life (3). Thus, this result constitutes further evidence that the fatigue theory is in essential agreement with the behavior of sack paper in repeated tension.

It may also be noted from Table VI or Fig. 8 that at applied strains above the endurance limit the theory underestimated the true fatigue life in three of the four comparisons. This underestimation was anticipated inasmuch as the method selected to arrive at an average reload slope was such as to lead to conservative estimates, as discussed earlier in this report.

COMPARISON OF TOTAL STRETCH AND VIRGIN STRETCH

One of the critical assumptions employed in formulating the fatigue life theory of sack paper is that the rupture point of the tensile load-elongation curve is an invariant with respect to repeated tension processes (3). That is, with reference to Fig. 9, it is assumed that the tensile strength, T ,

and the total stretch, e_t , at the point of rupture, R , after repeated tension are the same as the virgin strength and virgin stretch. This assumption was adopted in the spirit of a first approximation. It is appropriate, therefore, to re-examine this approximation whenever possible for the purpose of validating that aspect of the theory or to suggest further refinement of the theory.

Inasmuch as the present form of the fatigue life theory is oriented to an applied strain type of repeated tension (3), the invariancy of the total stretch is critical to the equation for fatigue life. Review of the derivation of this equation will reveal that the focal point of the analysis is determination of the number of strain applications at which the sum of the nonrecoverable stretch and the applied strain exceeds the total stretch. By assuming that the total stretch is the same as the virgin stretch, it is possible to predict fatigue life.

A review of the very considerable amount of repeated tension testing performed in connection with Project 2033 enables a reasonably comprehensive examination of the assumption in question. First, it should be mentioned that quite frequently the total strain at rupture after repeated tension is less than the total strain at the peak of the preceding cycle as illustrated in Fig. 10. That is, the specimen breaks during the loading before it attains the load and total strain which it was capable of sustaining during the preceding cycle. No explanation can be offered for the mechanism involved in this behavior and certainly the fatigue life theory (3) does not attempt to account for this occurrence.

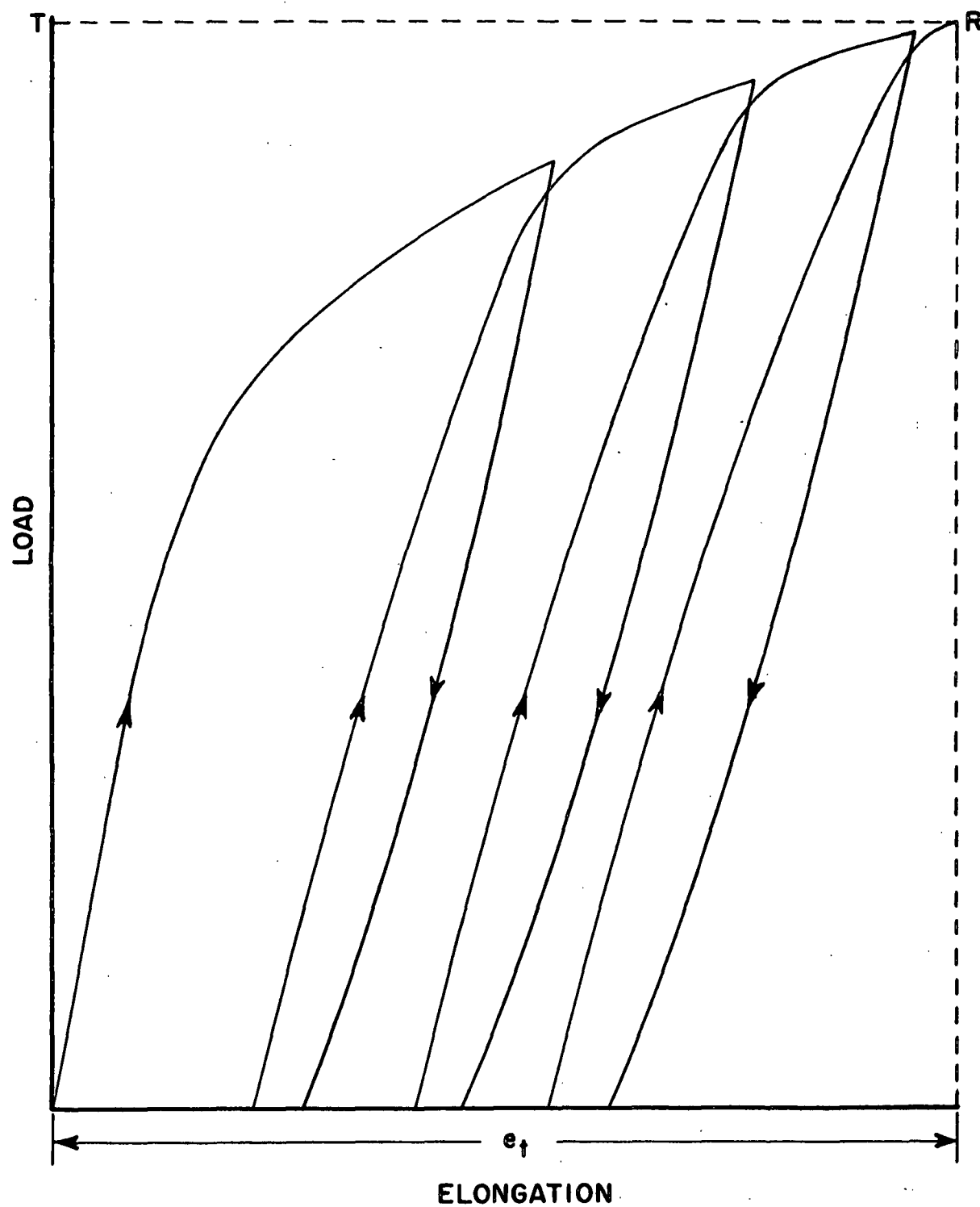


Figure 9. Total Stretch of Sack Paper in Repeated Tension

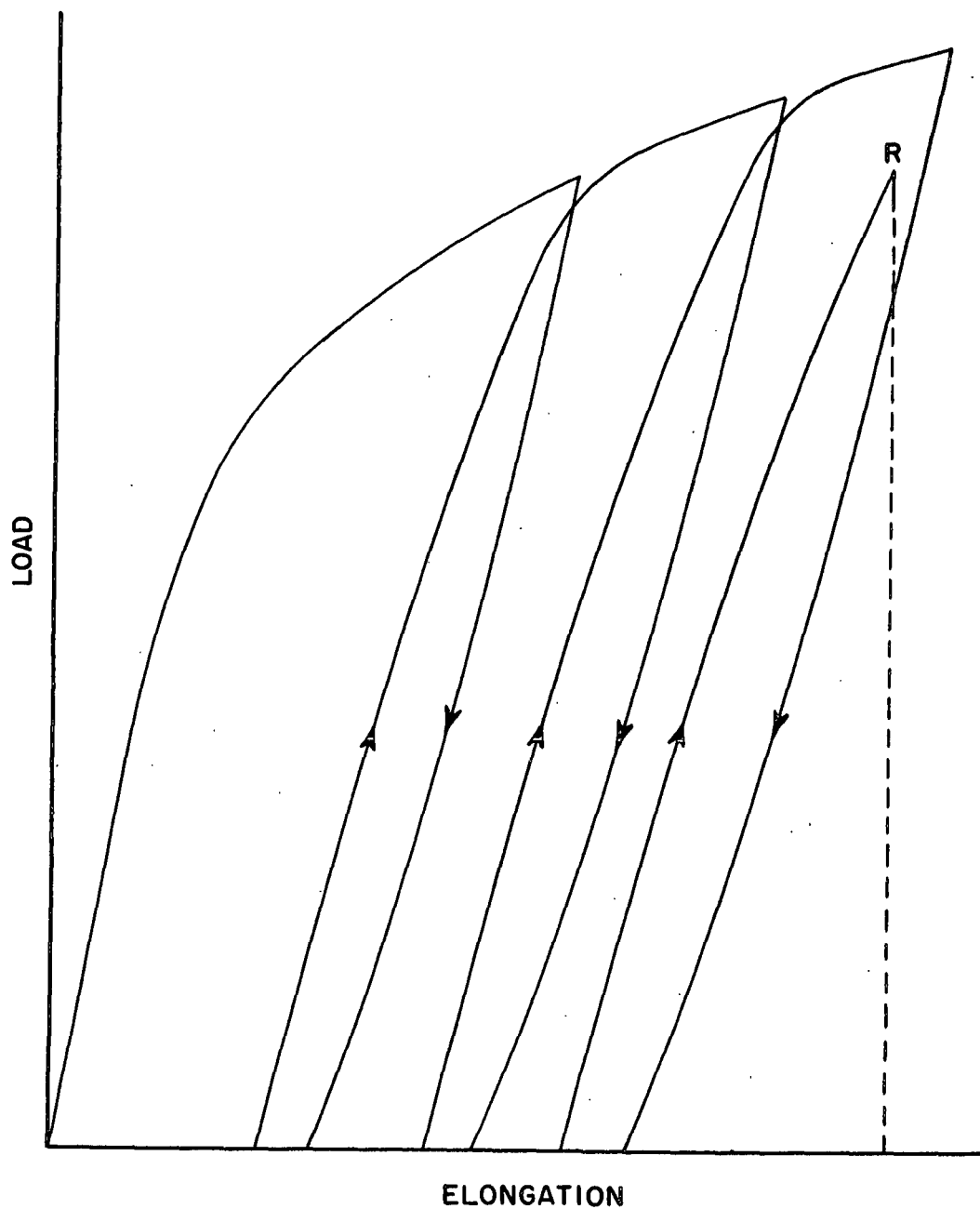


Figure 10. Illustration of Repeated Tension Rupture at Lesser Load and Strain Than Sustained in Last Safe Cycle

It is believed, however, that the aforementioned behavior is not of great import to the fatigue life equation inasmuch as in fatigue life one is concerned with the last safe cycle of repeated tension rather than with what occurs during the failure cycle. Whatever may occur in the failure cycle, the fact remains that the specimen was capable of sustaining a given peak load and strain during the last safe cycle. Accordingly, the assumption of invariancy of the rupture point in tension perhaps should be rephrased to state that "the peak load and total strain sustained during the last safe cycle is approximately the same as the virgin strength and virgin stretch." In the following investigation of total stretch, therefore, total stretch was taken as the greater of (a) the total strain of the last safe cycle, and (b) the total stretch at rupture.

A comparison of virgin stretch and total stretch after various numbers of strain cycles is afforded by the data of Table VIII. This table is divided into two parts--the first part concerning materials from the 1957 fabrication program and the second part involving regular and extensible papers from the recent second fabrication program. The latter data are the preliminary results of the Instron fatigue life tests which are to be compared with progressive height, sack impact performance. The samples listed in Table VIII as "first fabrication program" were tested in connection with (a) Reference (2), (b) the present investigation, and (c) certain allied exploratory studies not formally reported elsewhere.

As an example of the comparison of total and virgin stretch afforded by Table VIII, consider the first entries regarding Sample R tested in the machine direction. The virgin tensile test is denoted as zero cycles and

TABLE VIII

COMPARISON OF TOTAL STRETCH OF REPEATED TENSION AND VIRGIN STRETCH

Sample	Description of Sack Paper	Direction	Type of Applied Strain Process	No. of Specimens	No. of Cycles	Total Stretch, %	Difference, % ^a
First Fabrication Program							
R	50-lb. regular kraft	In	--	20	0	1.83	--
			--	10	1	1.80	-1.5
			Constant	10	2	1.81	-1.0
			Progressive	10	5	1.72	-6.0
			Constant	10	6	1.71	-6.5
			Progressive	10	13	1.70	-7.0
			Constant	10	20	1.61	-12.0
			Av.				-5.7
R	50-lb. regular kraft	Cross	--	20	0	2.57	--
			--	10	1	2.72	+6.0
			Constant	10	3	2.50	-2.5
			Progressive	10	5	2.38	-7.5
			Constant	10	5	2.52	-2.0
			Progressive	10	9	2.48	-3.5
			Constant	10	13	2.64	+2.5
			Av.				-1.2
T	50-lb. regular kraft	In	--	20	0	1.33	--
			--	10	1	1.25	-6.0
			--	10	1	1.45	+9.0
			Constant	10	2	1.18	-11.0
			Constant	10	5	1.21	-9.0
			Progressive	10	6	1.35	+1.5
			Progressive	10	9	1.20	-9.5
			Av.				-4.2
T	50-lb. regular kraft	Cross	--	20	0	5.49	--
			--	10	1	5.75	+5.0
			Constant	10	4	5.75	+5.0
			Progressive	10	7	5.09	-7.0
			Progressive	10	11	4.55	-17.0
			Constant	10	11	5.18	-5.5
			Constant	10	20	4.80	-12.5
			Av.				-5.3
Composite Av.						-4.1	
A	50-lb. regular kraft	In	--	30	0	1.27	--
			--	12	1	1.24	-2.5
			Constant	15	2	1.26	-1.0
			Av.				-1.8
A	50-lb. regular kraft	In	--	30	0	1.27	--
			--	10	1	1.25	-1.5
			Constant	10	2	1.27	0.0
			Constant	10	4	1.30	+2.5
			Constant	9	14	1.25	-1.5
			Av.				-0.1
A	50-lb. regular kraft	In	--	30	0	1.27	--
			--	10	1	1.27	0.0
			Constant	10	2	1.25	-1.5
			Constant	10	4	1.17	-8.0
			Constant	10	14	1.30	+2.5
			Av.				-1.8
A	50-lb. regular kraft	Cross	--	20	0	4.56	--
			--	10	1	4.50	-1.5
			Constant	10	2	4.25	-7.0
			Constant	15	3	4.45	-2.5
			Av.				-3.7
A	50-lb. regular kraft	Cross	--	20	0	4.56	--
			--	10	1	4.28	-6.0
			Constant	10	2	4.61	+1.0
			Constant	10	4	4.35	-4.5
			Constant	10	14	4.25	-7.0
			Av.				-4.1
Composite Av.						-2.3	

^a Based on virgin stretch.

TABLE VIII--Continued

COMPARISON OF TOTAL STRETCH OF REPEATED TENSION AND VIRGIN STRETCH

Sample	Description of Sack Paper	Direction	Type of Applied Strain Process	No. of Specimens	No. of Cycles	Total Stretch, %	Difference, % ^a
<u>Second Fabrication Program</u>							
HH	50-lb. Regular kraft	In	-- Progressive	6 6	0 3	1.4 1.3	-- -7
HH	50-lb. Regular kraft	Cross	-- Progressive	6 6	0 4	2.8 3.0	-- +7
JJ	50-lb. Regular kraft	In	-- Progressive	6 6	0 4	1.6 1.4	-- -12
JJ	50-lb. Regular kraft	Cross	-- Progressive	6 6	0 5	3.6 4.1	-- +14
SS	50-lb. Extensible kraft	In	-- Progressive	6 6	0 10	6.3 5.8	-- -8
SS	50-lb. Extensible kraft	Cross	-- Progressive	6 6	0 6	4.3 4.7	-- +9
PP	50-lb. Extensible kraft	In	-- Progressive	6 6	0 16	12.4 11.8	-- -5
PP	50-lb. Extensible kraft	Cross	-- Progressive	6 6	0 7	5.4 5.6	-- +4
In-Machine Composite Average							-8.0
Cross-Machine Composite Average							+8.5
Over-all Composite Average (All comparisons)							-2.8

^a Based on virgin stretch.

the total stretch in this instance is simply the virgin stretch. Based on tests of 20 specimens, the average virgin stretch was 1.83%. When ten specimens were subjected to one cycle of applied strain and then loaded to failure (that is, the single-cycle test described earlier in this report) the average total stretch was 1.80%, a decrease of 1.5% from the virgin stretch. When 10 specimens were subjected to two cycles of strain, the total stretch was 1.81% on the average-- a decrease of 1.0% from the virgin test. This particular set of tests was one of the fatigue life tests described earlier in this report.

The fourth line of data pertains to an allied study where the applied strain was progressively increased and the specimens ruptured after five cycles, on the average. For these specimens the average total stretch was 1.72%, a decrease of 6% from the virgin stretch. Continuing to the remaining higher numbers of cycles, this sample of sack paper exhibited progressively declining total stretch, differing by 12% from the virgin stretch when 20 cycles were applied to the paper.

This trend is not in evidence with the cross direction of Sample R, however. Although a decrease of 7.5% occurred after five cycles, the total stretch after thirteen cycles was actually greater than the virgin stretch by 2.5%. Similarly, with Sample T, machine direction, there are some rather large differences between virgin and total stretch, but there is no orderly trend for total stretch to change with the number of cycles. By contrast, the cross-direction tests on Sample T again suggest a trend to lower total stretch as the number of cycles increases.

Turning attention to Sample A which was tested in connection with Reference (2), it may be seen that the total stretch appears to be independent

of the number of cycles and generally about equal to the virgin stretch, although differences as great as 8% occurred.

The samples from the second fabrication program were tested with a progressively increasing applied strain process. Rather large differences were noted between virgin stretch and total stretch--as great as 14%--representing both increases and decreases relative to virgin stretch. It may be noted that in all instances the total stretch decreased in the machine-direction tests and increased in the cross-direction tests. It should also be noted that the sample size in these tests was only six rather than ten as with most of the preceding comparisons.

A number of average differences and (weighted) composite average differences are listed in Table VIII. These have been computed by retaining the algebraic sign of the individual differences because it was of interest to discern whether total stretch varies in some progressive manner as the number of cycles increases (rather than randomly due to sampling variability). On the basis of averages the total stretch varies rather modestly about the virgin stretch. With Samples R and T the total stretch was 4.1% less than the virgin stretch. With Sample A, the total stretch decreased 2.3% from the virgin stretch, on the average. The samples from the second fabrication program exhibited differences of about 8%, on the average, with the sense of the difference depending on the direction of test.

Taken in their entirety, it is difficult with these data to perceive any systematic trend for total stretch to depend on the number of strain cycles, a result which is in agreement with the literature (12-14). The general result appears to be a slight decrease in stretch due to cycling but not exceeding, say,

about 3% on the average. Individual differences, of course, were much more severe with decreases in stretch as great as 17% and increases as great as 14%. Differences of this magnitude, however, are not unusual with stretch measurements and one perhaps should be cautious about inferring that they are significant effects.

In summary, it is believed that there is no reason to doubt the validity of the assumption that the total stretch is essentially the same as the virgin stretch in the sense of a first approximation. Rather large differences may occur, however, due to limited sampling because of the inherently high variability in stretch of paper.

If more extensive investigation revealed that the total stretch in repeated tension differs significantly from the virgin stretch, refinement of the fatigue theory to account for the effect is possible. Suppose that it were established that the total stretch is proportional, but not equal, to the virgin stretch within the range of cycles of interest, that is, $\underline{e}_t = \gamma \underline{e}_v$. For example, if the total stretch were typically 95% of the virgin stretch, the factor γ would be 0.95. The effect of this refinement in the theory of fatigue life in Reference (3) is that the right-hand side of Equation (12) therein becomes $\gamma \underline{e}_v$, rather than \underline{e}_v . When the remainder of the derivation is carried out, it will be found that the equation for fatigue life [Equation (25) of Reference (3) or Equation (1) of this report] becomes:

$$N = \frac{\log \left\{ \frac{(\underline{e}_a/\underline{e}_v) - \gamma \alpha - (\beta - \alpha)(\underline{e}_o/\underline{e}_v)}{(1 - \alpha)(\underline{e}_a/\underline{e}_v) - (\beta - \alpha)(\underline{e}_o/\underline{e}_v)} \right\}}{\log (1 - \alpha)} \quad (6)$$

This equation differs from the earlier form only in the inclusion of γ in one term in the numerator. It may also be shown that the equation for the theoretical endurance limit, e_{∞} , becomes

$$e_{\infty} = \gamma \alpha e_v + (\beta - \alpha) e_o \quad (7)$$

which differs from an earlier equation by the factor γ in the first term on the right-hand side. (One undesirable aspect of the refinement given by Equation (6) is that the theoretical fatigue life no longer becomes zero when the applied strain equals the virgin stretch; that is, the equation no longer describes the virgin tensile test, unless γ is set equal to unity for this special case.)

Table VIII reveals that several rather extreme differences between total stretch and virgin stretch occurred in the samples employed in the present study of fatigue life. Presumably these differences were due to sampling. Nonetheless, they can be expected to influence the accuracy of the predictions of fatigue life which were made earlier in this report. It was of interest, therefore, to correct the predicted values of fatigue life in accordance with the actual total stretch exhibited by the specimens during fatigue testing. This procedure relieves the prediction from dependency on the assumption of invariance of stretch and may give a clearer picture of whether the mechanism of repeated tension in other respects is adequately described by the fatigue life equation.

For this purpose, revised estimates of fatigue life were made using Equation (6) with an average value of γ determined from the fatigue tests. This calculation was performed for the machine-direction tests of Samples R

and T inasmuch as both showed several extreme differences between virgin stretch and total stretch and furthermore because the previously discussed agreement between theory and experiment was quite poor for Sample T in the machine direction. The revised predictions are listed in Table IX along with the original predictions and the experimental fatigue lives.

TABLE IX

COMPARISON OF REVISED PREDICTION OF FATIGUE LIFE (CORRECTED FOR TOTAL STRETCH) AND OBSERVED FATIGUE LIFE

Sample	Direction	Applied Strain, %	γ^a	Fatigue Life		Observed
				Theoretical Original	Revised	
R	In	1.30	0.918	2	1	2
		1.08		4	3	6
		1.01		∞	5	16
		0.90		∞	∞	> 20
T	In	1.07	0.910	2	1	1
		0.93		5	3	2.5
		0.90		∞	4	5

^a $\gamma = (\text{Total Stretch} / \text{Virgin Stretch})$

It may be seen in Table IX that the revised estimates in the case of Run R were somewhat more conservative than the original predictions. The revised theoretical endurance limit in this case is a strain of 0.96%. Figure 11 presents a curve of the revised theoretical fatigue life and also shows the

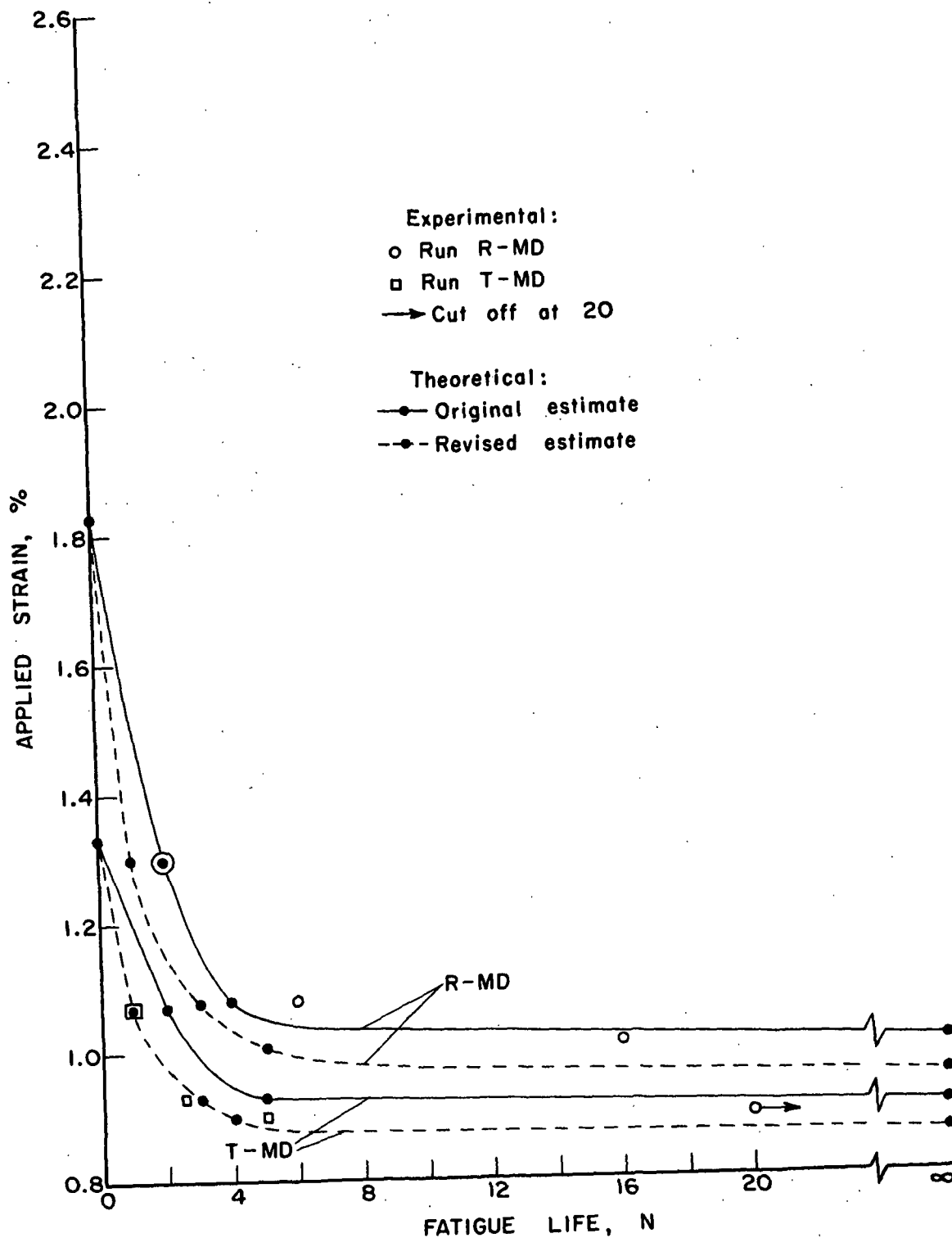


Figure 11. Graph of Theoretical Fatigue Life Showing Revised Estimates
Based on Correction for Total Stretch

original prediction as a reference along with the experimental results. It may be seen that, while the revised estimates are quite conservative, the shape of the fatigue life curve appears to agree with the experimental points and the theoretical endurance limit appears reasonable.

In the case of Run T, a marked improvement was made in the estimates by accounting for the actual total stretch of the paper. Whereas the original predictions grossly overestimated the fatigue life near the endurance limit, the revised estimates are in very good agreement with experiment. The revised estimate of the endurance limit is a strain of 0.87%. Inspection of Fig. 11 suggests that this estimate of endurance limit is reasonable in view of the experimental points.

It appears, therefore, that the discrepancy between theoretical and observed fatigue life for the machine direction of Run T, which was discussed earlier in this report, was attributable to the low total stretch exhibited by the paper during the experimental fatigue tests. After correcting the analysis for the low stretch, the fatigue life theory adequately described the repeated tension behavior of these test specimens.

Comparison of the original and revised estimates of fatigue life emphasizes the sensitivity of fatigue life to small changes in total stretch--changes on the order of 10%--and the need, therefore, for adequately large sampling of the materials in fatigue studies.

PROPOSALS FOR FUTURE WORK

In view of the progress which has been made in developing an understanding of fatigue behavior of sack paper, perhaps the most compelling next step in this general area of research is investigation of the relationship between sack impact performance and the uniaxial fatigue performance of the sack paper from which the sacks are fabricated. A sufficiently precise relationship of this type, in conjunction with the theory for the fatigue life of the paper, would permit relating sack performance directly to the uniaxial tension load-elongation properties of the sack paper. As noted in the Introduction, this study is in progress for the sacks of the second fabrication program.

It should be mentioned that the fatigue theory was initially conceived in terms of regular rather than extensible sack paper because (a) there was more extensive experience in repeated tension behavior available for regular papers, and (b) it was known that a straight-line approximation to the virgin tension curve of regular paper was feasible, while it was uncertain for extensible paper. Preliminary inspection, however, of the virgin and repeated tension behavior of extensible papers from the second fabrication program indicates that there is every reason to expect the theory can also be applied to extensible papers. Specifically, the critical assumption of invariancy of the total stretch appears to be valid, as discussed elsewhere in this report. And, furthermore, the virgin load-elongation curves of extensible papers appear to be amenable to straight-line approximation, although not as well as regular papers. Thus, it is believed that no new considerations will be required to apply the fatigue theory to extensible papers.

Experiments have shown that the paper in a sack is subjected to biaxial stresses and strains during impact of the sack. On intuitive grounds, therefore, it is anticipated that biaxial stress-strain properties of sack paper should be more closely related to sack performance than are uniaxial properties. A desirable next step in the over-all analysis of fatigue performance, therefore, is an experimental and theoretical investigation of the relationship between biaxial fatigue of sack paper and sack impact performance. In particular, this study will require development of an adequate experimental method of evaluating biaxial fatigue life. Work on the design of a test apparatus is in progress. If a significant improvement in predicting sack performance were forthcoming by this approach, the ensuing research efforts could be directed to one or both of the following alternatives: (a) development of a biaxial fatigue test suitable for use by the sack and sack paper industry, and (b) utilize a classical approach of developing the relationship between biaxial and uniaxial fatigue life of sack paper, thereby permitting use of the test equipment and theory of uniaxial tension for evaluating the potential performance of sack paper.

Looking to future work more closely allied to the current theory of uniaxial fatigue, it is believed that there are several profitable lines of research, as described in the following paragraphs.

FURTHER VERIFICATION OF THEORY

It would be desirable to augment the current verification with additional samples of sack paper--regular and extensible--representing other levels of virgin and repeated tension properties, if for no other reason than to broaden

the base of experimental data on which to judge the adequacy of the theory. The aforementioned testing now going forward on Instron fatigue of the fabrication program materials provides an opportunity for this additional verification.

IMPROVEMENT IN ACCURACY OF THEORY

The present study demonstrated that the theory and experiment were in reasonable agreement on (a) the shape of the fatigue life vs. applied strain curve and (b) the magnitude of the endurance limit. There were rather large numerical differences, however, between predicted and observed fatigue life in some instances, probably because of the sensitivity of fatigue life to small differences in applied strain near the endurance limit.

Very likely the differences between theory and experiment are due in part to the errors of approximation that are necessarily incurred by representing the load-elongation curve by two straight lines. A more exact method of approximation has been developed in connection with Project 2033 (15); it is an adaptation for sack paper of a method originally devised for metals (16) and gives a close curvilinear approximation throughout the entirety of the load-elongation curve. Application of this refinement to the study of the materials of the present investigation demands no further experimental work but only analytical work. It cannot be expected, however, that this method of approximation will lead to any simplification in the mathematical aspects of the theory and actually may be somewhat more complicated, although the latter may be justified by an improvement in accuracy.

EXTENSION OF FATIGUE THEORY TO OTHER REPEATED TENSION PROCESSES

Although the present form of the fatigue life equation is concerned with repetitive application of a constant magnitude of strain (which presumably is closely allied to the constant height sack impact test), the underlying principles are believed to be applicable to the numerous other conceivable repeated tension processes. Theoretical development of the fatigue life equation is in progress for another type of process, namely, a progressively increasing applied strain (specifically, arithmetic progression of applied strain) which bears an analogy to the progressive height sack impact test. This version of the theory is appropriate to the current Instron fatigue evaluation of sack papers from the second fabrication program.

In view of the concept of sack impact as an applied energy process, it is recommended that the theory be developed for the cases of constant and progressively increasing applied energy. No particularly formidable obstacles to the theoretical development are foreseen and it is expected that the fatigue life equations will parallel their applied strain counterparts. Previous work in this laboratory (2) and by Mappus (6) have demonstrated the feasibility of experimentally studying applied energy processes by means of an Instron testing machine.

SIMPLIFICATION OF FATIGUE LIFE EQUATION

It appears quite possible that simplification in the fatigue life equation [Equation (1)] can be effected which will make the theory more readily applicable in the mill and sack plant. This simplification is in progress and hinges on the possibility that there may be some empirically

determined interrelationships between the tension load-elongation parameters entering into the fatigue life equation, such that one or more of the parameters may be expressed in terms of other parameters and thereby eliminated from the fatigue life equation. For example, preliminary investigation suggests that the proportional limit strain, ϵ_o , and the reload slope, $\underline{S_r}$, possibly may be correlated with the remaining fatigue parameters ($\underline{e_y}$, $\underline{S_o}$, and $\underline{S_p}$) within certain classes of sack papers and thus the former may be eliminated from the fatigue life equation. In this event the fatigue theory could be presented as a family of curves of fatigue life versus applied strain for each class of paper so correlated, avoiding thereby the necessity of numerically evaluating the fatigue life equation in applications. Moreover, the single-cycle tensile test would not be required--only the virgin test for the purpose of finding the elastic and plastic slopes and the virgin stretch.

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